GENERATION RULES FOR ANY COMPLETE FACTORIAL DESIGN OF THE ANALYSIS OF VARIANCE

Notation

The application of generation rules necessitates the consistent use of some notation system in which the structural definition of the design to be elaborated is expressed. Generally, the choice of a notation system is arbitrary, and any preferences are mainly based on criteria of economy. The system to be presented has been adapted from that of R. G. D. Steel & J. H. Torrie (*Principles and procedures of statistics*. New York: McGraw-Hill, 1960) and that of J. L. Myers (*Fundamentals of experimental design*. Boston: Allyn and Bacon, 1966). The notation has been modified to avoid unnecessary complications in the rules. The suggested system, then, although descriptively equivalent to alternative systems, is preferred over others primarily because it best accomodates the generation rules.

Factors

Factors are denoted by uppercase letters, i.e., A, B, C, . . . , Z. To accomodate the fact that sampling in psychology and communications research typically applies to experimental *subjects* rather than events in a more general sense, the use of the letter S to designate subjects is recommended.

The designation of the sampled entities, S (or possibly E), precedes all factor designations in the structural definition.

If entities in S are completely randomized through some factor, say A, the associated measures are referred to as *independent*. An independent-measure factor is denoted by parentheses, e.g., S(A).

If measures are repeatedly taken on the same entities in S through some factor, say A, the measures are referred to as *repeated*. A repeated-measure factor is denoted indirectly by not being parenthesized.

For the generation of expected mean squares associated with the various sources of variation in a particular design, it is necessary to indicate random and fixed factors in the structural definition. A *random* factor is denoted by a prime following the factor-designating letter, e.g., S'. A *fixed* factor is denoted indirectly by the absence of a prime.

Levels

Levels are denoted by lower-case letter subscripts corresponding to the factor letter, e.g., A_a, . . . , S_s, . . . , Z_z. In order to designate the nesting of subjects (or events) under any particular factor or factors, all subscripts associated with parenthesized factors, that is, with independent-measure factors, are put down with S_s, forming a multiple subscript, e.g., S_{sabc}.

The present outline of rules applies only to designs in which the number of subjects (or events) in the s levels of S is identical. Adjustments for unequal-n designs can easily be made, however.

The structural definition

In general, in psychology and communications research, subjects (or events) will be dealt with as a random factor. Thus, typically, but not necessarily, the structural definition will begin with S'_s rather than with S_s. For pure repeated-measure designs, factor and level designators are simply added on, e.g., $S'_sA_aB_bC_c...Z_z$. The only difference in pure independent-measure designs is that all subscripts within parentheses are added onto S'_s, e.g., $S'_{sabc}(A_aB_bC_c)$. In designs which contain both repeated- and independent-measure factors, so-called "mixed" designs, the analogous operations are executed, e.g., $S'_{sab}(A_aB_b)C_cD_d$, where A and B are independent-, and C and D are repeated-measure factors. To provide an easier overview, it is suggested but not required that, as in the latter example, alle independent-measure factors be blocked together after $S'_{s...}$ and followed by any repeated-measure factors. [If, however, different arrangements are preferable for some reason, they can be used, e.g., $S'_{sab}(A_a)C_c(B_b)D_d$.]

The General Plan for Generating a Particular Design

After the *structural definition* of a particular design has been specified, the generation of the entire design proceeds in stages.

1. The *sources of variation* are determined from the structural definition. A table is constructed, containing all the elements into which the total variation is partitioned.

2. For each source of variation, an expression defining its *degrees of freedom* is generated from the notation of the particular source under consideration.

3. For each source of variation, *algebraic* and/or *computational formulas* for the sums of squares are generated from the expression defining the degrees of freedom.

4. For each source of variation, the *expected mean square* is generated from the table of the partitioned variation.

5. The *proper error terms* for all *F ratios* are determined.

Determining the Sources of Variation

Rule 1a.

Produce all possible combinations of the factor letters in the design. Start by sampling one out of the k letters, and end by sampling k out of the k letters. For simplicity, put all combinations in alphabetical order.

The resulting number of sources thus is $\sum_{i=1}^{k} {}_kC_i$.

Illustration: For $S'_sA_aB_b$ or $S'_{sa}(A_a)B_b$ or $S'_{sab}(A_aB_b)$, $k = 3$.

Consequently, all cases yield the same set of combinations: A, B, S, AB, AS, BS, ABS.

Rule 1b.

Maintain all subscripts as specified in the structural definition. Also, maintain any primes.

Illustration: For the design $S'_{sa}(A_a)B'_b$, the chain A_a, B'_b, S'_{sa}, $A_aB'_b$, $A_aS'_{sa}$, $B'_bS'_{sa}$, $A_aB'_bS'_{sa}$ results.

Rule 1c.

Eliminate those combinations which have any subscript letters not contained in the factor letters of the combination.

Illustration: In the design described above, the combinations S'_{sa} and $B'_bS'_{sa}$ are eliminated because they contain subscript $_a$, but not factor letter A.

Rule 1d.

Transpose combinations which contain multiple subscripts by letting $S'_{s\ldots}$ precede other factor denotations. Parenthesize the factor designations defined in the subscripts following $_s$ of S'.

Illustration: In the design described above, the combination $A_aS'_{sa}$ changes to $S'_{sa}(A_a)$, and $A_aB'_bS'_{sa}$ changes to $S'_{sa}(A_a)B'_b$.

Simplification: The execution of rules 1a through 1d yields all the sources of variation associated with a particular design. Since the factor letters define these sources, and the subscripts are not used in the application of subsequent rules, the notation may be reduced to factor letters, primes, and parentheses.

For the conventional, complete description of the sources of variation, the factor-letter combinations, written as subscripts, are preceded by σ^2, which, in turn, is preceded by all remaining factor letters of the design. Again conventionally, these remaining letters, taken from the structural definition, are written in lower case, e.g., for the design $S'_{sab}(A_aB_b)C_c$, the source AC is written $bs\sigma^2_{AC}$.

Generating Degrees of Freedom

The generation of degrees of freedom from the factor-letter combination denoting a particular source proceeds in stages similar to those used in the determination of the sources of variation of a design: first, the combinations are exhausted, and second, omission rules are applied. It should be noted, however, that the omission rules can be applied as the combinations are determined. The elimination of terms during the process of generation may be considered more economical than the exhaustive generation of terms which are subsequently tested for inclusion-exclusion. Thus, generation and omission rules may be applied quasi-simultaneously (that is, immediately after a particular combination is determined, inclusion or exclusion is decided upon). However, for clarity, the rules are presented for the exhaustive generation of terms followed by the execution of omission decisions.

Rule 2a.

Assign positive value to the combination of all factor letters defining the source of variation. This combination constitutes the first term in the <u>df</u> string to be generated.

Table 1
The Determination of Sources of Variation
Illustrated with Various Designs
(Initial Stages)

	Combinations*	Elimination of improper sources#		
Design	$S'_sA_aB_bC_c$	$S'_{sa}(A_a)B_bC_c$	$S'_{sab}(A_aB_b)C_c$	$S'_{sabc}(A_aB_bC_c)$
Sources of variation	A_a	A_a	A_a	A_a
	B_b	B_b	B_b	B_b
	C_c	C_c	C_c	C_c
	S'_s	~~S'_{sa}~~#	~~S'_{sab}~~#	~~S'_{sabc}~~#
	A_aB_b	A_aB_b	A_aB_b	A_aB_b
	A_aC_c	A_aC_c	A_aC_c	A_aC_c
	$A_aS'_s$	$A_aS'_{sa}$¢	~~$A_aS'_{sab}$~~	~~$A_aS'_{sabc}$~~
	B_bC_c	B_bC_c	B_bC_c	B_bC_c
	$B_bS'_s$	~~$B_bS'_{sa}$~~	~~$B_bS'_{sab}$~~	~~$B_bS'_{sabc}$~~
	$C_cS'_s$	~~$C_cS'_{sa}$~~	~~$C_cS'_{sab}$~~	~~$C_cS'_{sabc}$~~
	$A_aB_bC_c$	$A_aB_bC_c$	$A_aB_bC_c$	$A_aB_bC_c$
	$A_aB_bS'_s$	$A_aB_bS'_{sa}$¢	$A_aB_bS'_{sab}$¢	~~$A_aB_bS'_{sabc}$~~
	$A_aC_cS'_s$	$A_aC_cS'_{sa}$¢	~~$A_aC_cS'_{sab}$~~	~~$A_aC_cS'_{sabc}$~~
	$B_bC_cS'_s$	~~$B_bC_cS'_{sa}$~~	~~$B_bC_cS'_{sab}$~~	~~$B_bC_cS'_{sabc}$~~
	$A_aB_bC_cS'_s$@	$A_aB_bC_cS'_{sa}$@¢	$A_aB_bC_cS'_{sab}$@¢	$A_aB_bC_cS_{sabc}$@¢

* Rule 1a. # Rule 1c. @ Rule 1b. ¢ Rule 1d to be applied.

Table 2
The Determination of Sources of Variation
Illustrated with Various Designs
(Final Stage)

Design	$S'_s A_a B_b C_c$	$S'_{sa}(A_a)B_b C_c$	$S'_{sab}(A_a B_b)C_c$	$S'_{sabc}(A_a B_b C_c)$
Sources of variation	$bcs\sigma^2_A$	$bcs\sigma^2_A$	$bcs\sigma^2_A$	$bcs\sigma^2_A$
	$acs\sigma^2_B$	$acs\sigma^2_B$	$acs\sigma^2_B$	$acs\sigma^2_B$
	$abs\sigma^2_C$	$abs\sigma^2_C$	$abs\sigma^2_C$	$abs\sigma^2_C$
	$abc\sigma^2_{S'}$			
	$cs\sigma^2_{AB}$	$cs\sigma^2_{AB}$	$cs\sigma^2_{AB}$	$cs\sigma^2_{AB}$
	$bs\sigma^2_{AC}$	$bs\sigma^2_{AC}$	$bs\sigma^2_{AC}$	$bs\sigma^2_{AC}$
	$bc\sigma^2_{AS'}$	$bc\sigma^2_{S'(A)}$*		
	$as\sigma^2_{BC}$	$as\sigma^2_{BC}$	$as\sigma^2_{BC}$	$as\sigma^2_{BC}$
	$ac\sigma^2_{BS'}$			
	$ab\sigma^2_{CS'}$			
	$s\sigma^2_{ABC}$	$s\sigma^2_{ABC}$	$s\sigma^2_{ABC}$	$s\sigma^2_{ABC}$
	$c\sigma^2_{ABS'}$	$c\sigma^2_{S'(A)B}$*	$c\sigma^2_{S'(AB)}$*	
	$b\sigma^2_{ACS'}$	$b\sigma^2_{S'(A)C}$*		
	$a\sigma^2_{BCS'}$			
	$\sigma^2_{ABCS'}$	$\sigma^2_{S'(A)BC}$*	$\sigma^2_{S'(AB)C}$*	$\sigma^2_{S'(ABC)}$*

* Rule 1d.

Illustration: The first term for the source S(A)B is sab, or, since it denotes a product, any permutation thereof, e.g., abs.

Rule 2b.

Reduce the complexity of the last letter combination generated by unity, and change the prefix to the opposite mode (from + to –, or vice versa).

Illustration: If +sab defines the last term generated, the complexity is reduced to combinations of two letters, all having negative value.

Rule 2c.

At the determined complexity, generate all combinations of the letters specified in the source-defining combination.

Illustration: For the source S(A)B, the two-letter combinations are –ab, –as, –bs.

Rule 2d.

If the source-defining letter combination contains parenthesized letters, eliminate all generated combinations which do not contain all the parenthesized letters.

Illustration: Given the source S(AB)C, the resulting triplets are –abc, –abs, –acs, –bcs. From this set –acs and –bcs are dropped.

Rule 2e.

Continue to apply rules (*2b*) through (*2d*)

(a) if the source-defining combination does *not* contain parentheses, until the complexity zero is reached, at which time unity (dependent upon the prefix changes, either + or –) is put down, terminating the df string generated; or

(b) if the source-defining combination contains parentheses, until the complexity of the parenthesized subset is reached, at which time all parenthesized letters (either + or –) are put down, terminating the df string generated.

Illustration *(a)*: For the source ABC, the string abc–ab–ac–bc+a+b+c–1 results.

Table 3

The Generation of Degrees of Freedom

Illustrated with Various Sources of Variation

Source of variation	String defining <u>df</u> s
. . . σ^2_{A}	(a*−1#$) a−1
. . . σ^2_{AB}	(ab*−a#−b@+1#$) ab−a−b+1
. . . σ^2_{ABC}	(abc*−ab#−ac@−bc@+a#+b@+c@−1#$) abc−ab−ac−bc+a+b+c−1
. . . $\sigma^2_{S'(A)}$	(as*−a#−~~s~~¢&) as−a
. . . $\sigma^2_{S'_{sa}(A_a)B_b}$	(abs*−ab#−as@−~~bs~~¢+a#&) abs−ab−as+a
. . . $\sigma^2_{S'_{sab}(A_aB_b)C_c}$	(abcs*−abc#−abs@−~~acs~~¢−~~bcs~~¢+ab#&) abcs−abc−abs+ab
. . . $\sigma^2_{S'_{sa}(A_a)B_bC_cD_d}$	(abcds*−abcd#−abcs@−abds−acds−~~bcds~~¢+abc# +abd@+abs+acd+acs+ads+~~bcd~~¢+~~bcs~~¢+~~bds~~¢+~~cds~~¢ −ab#−ac@−ad−as−~~bc~~¢−~~bd~~¢−~~bs~~¢−~~cd~~¢−~~cs~~¢−~~ds~~¢+a#&) abcds−abcd−abcs−abds−acds+abc+abd+abs+acd +acs+ads−ab−ac−ad−as+a

* Rule 2a. # Rule 2b. @ Rule 2c. ¢ Rule 2d. $ Rule 2e(<u>a</u>).
& Rule 2e(<u>b</u>).

Illustration *(b)*: For the source S(AB)C, the resulting string is abcs−abc−abs +ab.

Generating Algebraic and Computational Formulas for Sums of Squares

The formulas for each source of variation are determined in a one-to-one correspondence of terms from the string defining the degrees of freedom associated with this source.

(A) Sums of squares, algebraic.

Rule 3a (A).

Put all elements of a particular <u>df</u> string as subscripts. Between prefixes and subscripts, insert means composed of the same letters as the subscripts. Set $\overline{M}$, the grand mean, for unity subscripts.

Illustration: For source AB asssociated with the <u>df</u> string ab−a−b+1, $\overline{AB}_{ab}-\overline{A}_a-\overline{B}_b+\overline{M}$.

Rule 3b (A).

Parenthesize and square the expression.

Illustration: $(\overline{AB}_{ab}-\overline{A}_a-\overline{B}_b+\overline{M})^2$.

Rule 3c (A).

For every factor letter in the design, let a summation operator precede the parenthesized expression. Eliminate those operators associated with letters not defined in the subscripts within the parenthesized expression, and let their letters as constants, precede the remaining summation operators.

Illustration: Given the structural definition $S'_{sab}(A_aB_b)C_c$, and, for the source AB the expression $(\overline{AB}_{ab}-\overline{A}_a-\overline{B}_b+\overline{M})^2$, the expression first becomes $\sum^{a}\sum^{b}\sum^{c}\sum^{s}(\overline{AB}_{ab}-\overline{A}_a-\overline{B}_b+\overline{M})^2$, and then $cs\sum^{a}\sum^{b}(\overline{AB}_{ab}-\overline{A}_a-\overline{B}_b+\overline{M})^2$.

Note: Lower-case letters *as subscripts* denote variables. In contrast, the same letters *above summation signs* denote the number of levels of the associated factors, or the number of subjects (or events).

Table 4

The Generation of Algebraic SS Expressions Illustrated with Various Sources of Variation

Source of variation	Degrees of freedom and sums of squares
$bs\sigma^2{}_A$	df = a −1 SS = $bs\sum^{a@} (\overline{A}_a{}^{*} - \overline{M}_1{}^{*})^{2\#}$
$s\sigma^2{}_{AB}$	df = ab −a −b +1 SS = $s\sum^{a}\sum^{b}(\overline{AB}_{ab}-\overline{A}_a-\overline{B}_b+\overline{M})^2$
$\sigma^2{}_{S'(AB)C}$	df = abcs −abc −abs +ab SS = $\sum^{a}\sum^{b}\sum^{c}\sum^{s}(\overline{ABCS}_{abcs}-\overline{ABC}_{abc}-\overline{ABS}_{abs}+\overline{AB}_{ab})^2$, where $\overline{ABCS}_{abcs} = X_{abcs}$
$d\sigma^2{}_{S'(ABC)}$	df = abcs −abc SS = $d\sum^{a}\sum^{b}\sum^{c}\sum^{s}(\overline{ABCS}_{abcs}-\overline{ABC}_{abc})^2$

* Rule 3a(A). # Rule 3b(A). @ Rule 3c(A).

(B) Sums of squares, computational.

Rule 3a (B).

Set all factor letters specified in the structural definition of the design as subscripts of X, defining the data matrix. For every subscript of X, let a summation operator precede X.

Illustration: In an $S'_{sab}(A_aB_b)C_c$ design the data matrix is X_{abcs}, and the complexity of a summation operations for X is $\Sigma\Sigma\Sigma\Sigma X$.

Rule 3b (B).

Maintaining all prefixes, every element of the df string is associated with the specified set of basic operations.

Illustration: The df string $ab-a-b+1$ associated with source AB in an $S_{sab}(A_aB_b)C_c$ design yields $\Sigma\Sigma\Sigma\Sigma X-\Sigma\Sigma\Sigma\Sigma X-\Sigma\Sigma\Sigma\Sigma X+\Sigma\Sigma\Sigma\Sigma X$.

Rule 3c (B).

For any particular set of operations on X, the letters specified in the associated element of the df string are put down in greatest proximity to X, determining the range of the summations. All permutations of these letters are permissible.

Illustration: The element $\Sigma\Sigma\Sigma\Sigma X$ associated with the df element ac becomes $\Sigma\Sigma\overset{a}{\Sigma}\overset{c}{\Sigma}X$ or $\Sigma\Sigma\overset{c}{\Sigma}\overset{a}{\Sigma}X$.

Rule 3d (B).

The letter-denoted summation signs and X are parenthesized, and the parenthesized expression is squared.

Illustration: In the example above, $\Sigma\Sigma\overset{a}{\Sigma}\overset{c}{\Sigma}X$ becomes $\Sigma\Sigma(\overset{a}{\Sigma}\overset{c}{\Sigma}X)^2$.

Rule 3e (B).

The remaining summation signs are denoted by the remaining letter subscripts of X. Again, all permutations are permissible.

Illustration: In the example above, $\sum^{b}\sum^{s}(\sum^{a}\sum^{c}X)^2$ or $\sum^{s}\sum^{b}(\sum^{a}\sum^{c}X)^2$ results. Or more explicitly expressed, it becomes, e.g., $\sum^{b}\sum^{s}(\sum^{a}\sum^{c}X_{abcs})^2$.

Rule 3f (B).

For each elementary expression, the product of the letters heading the summation signs within parentheses is put down as the denominator.

Illustration: $(\sum^{a}\sum^{b}\sum^{s}X)^2$ becomes $\frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$, $\sum^{a}(\sum^{b}\sum^{s}X)^2$ becomes $\frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs}$, and $\sum^{a}\sum^{b}(\sum^{s}X)^2$ becomes $\frac{\sum^{a}\sum^{b}(\sum^{s}X)^2}{s}$, whereas $\sum^{a}\sum^{b}\sum^{s}X^2$ remains unchanged.

Table 5

The Generation of Computational SS Expressions

Illustrated with Various Sources of Variation

Source of variation	Degrees of freedom and sums of squares
$bs\sigma^2_A$	df = $a \quad -1$ SS = $\frac{\sum^{b}\sum^{s\$}(\,^{\text{¢}}\sum^{a@}X_{abs})^{2\,*\,\text{¢}}}{a^{\&}} -^{\#} \sum^{a}\sum^{b}\sum^{s\$}(X_{abs})^{2\,\text{¢}\,*}$ $= \frac{\sum^{b}\sum^{s}(\sum^{a}X)^2}{a} - \sum^{a}\sum^{b}\sum^{s}X^2$
$s\sigma^2_{AB}$	df = $ab \quad -a \quad -b \quad +1$ SS = $\frac{\sum^{s}(\sum^{a}\sum^{b}X)^2}{ab} - \frac{\sum^{b}\sum^{s}(\sum^{a}X)^2}{a} - \frac{\sum^{a}\sum^{s}(\sum^{b}X)^2}{b} + \sum^{a}\sum^{b}\sum^{s}X^2$
$\sigma^2_{S'(AB)C}$	df = $abcs \quad -abc \quad -abs \quad +ab$ SS = $\frac{(\sum^{a}\sum^{b}\sum^{c}\sum^{s}X)^2}{abcs} - \frac{\sum^{s}(\sum^{a}\sum^{b}\sum^{c}X)^2}{abc} - \frac{\sum^{c}(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs} + \frac{\sum^{c}\sum^{s}(\sum^{a}\sum^{b}X)^2}{ab}$
$d\sigma^2_{S'(ABC)}$	df = $abcs \quad -abc$ SS = $\frac{\sum^{d}(\sum^{a}\sum^{b}\sum^{c}\sum^{s}X)^2}{abcs} - \frac{\sum^{d}\sum^{s}(\sum^{a}\sum^{b}\sum^{c}X)^2}{abc}$

* Rule 3a(B). # Rule 3b(B). @ Rule 3c(B). ¢ Rule 3d(B). $ Rule 3e(B). & Rule 3f(B).

Generating Expected Mean Squares

The rules to be presented have been adapted from S. L. Crump (The estimation of variance components in analysis of variance. *Biometrics Bulletin*, 1946, 2, 7-11) and from E. F. Schultz, Jr. (Rules of thumb for determining expectations of mean squares in analysis of variance. *Biometrics*, 1955, 11, 123-135).

Given the table of completely described sources of variation, the $\mathcal{E}(\underline{MS})$ for each source is determined by testing all sources associated with the design for inclusion-exclusion.

Rule 4a.

Put down the σ^2 term associated with the source under consideration.

Illustration: For the source $cs\sigma^2_{AB}$ put down $cs\sigma^2_{AB}$. However, to determine *F* ratios, it suffices to use some abbreviation, e.g., σ^2_{AB}, or AB only.

Rule 4b.

Ignore all sources which are denoted by fewer subscribed factor letters than are contained in the denotation of the source under consideration.

Illustration: If the source is $s\sigma^2_{ABC}$, all sources associated with one or two subscripted factor letters are ignored.

Rule 4c.

Ignore all sources which do not contain all the subscripted factor letters associated with the source under consideration.

Clearly, rule *4c* implies rule *4b*, which serves greater economy only. Rule *4b* may therefore be neglected.

Illustration: If source $ds\sigma^2_{ABC}$ is under consideration, source $as\sigma^2_{BCD}$, lacking A in the subscript, is ignored.

Rule 4d.

Ignore all sources which contain non-parenthesized subscripted factor letters denoting fixed factors not defined in the subscript of the source under consideration.

Illustration: If source $b\sigma^2_{S'(A)}$ is under consideration, source $\sigma^2_{S'(A)B}$ is ignored, whereas source $\sigma^2_{S'(A)B'}$ is not ignored.

Rule 4e.

Put down all remaining sources. Insert addition signs between all terms.

Illustration: For the source $bs\sigma^2_A$ in the design S'(AB'), associated with the sources $bs\sigma^2_{A'}$, $as\sigma^2_{B'}$, $s\sigma^2_{AB'}$, $\sigma^2_{S'(AB')}$, the resulting expression is $bs\sigma^2_A + s\sigma^2_{AB'} + \sigma^2_{S'(AB')}$ or, abbreviated, A + AB' + S'(AB').

Determining Valid F Ratios

For each source of variation in a design, the proper error term must be determined.

Rule 5a.

Ignore the first term in the $\mathcal{E}(\underline{MS})$ of the source under consideration, and find the source associated with an $\mathcal{E}(\underline{MS})$ which is identical to the remaining terms.

If no match can be found, a valid $\underline{F}$ ratio does not exist for the source under consideration.

Illustration: In the design described above, the $\mathcal{E}(\underline{MS})$ associated with the source $bs\sigma^2_A$, when its first term is ignored, reduces to $s\sigma^2_{AB'} + \sigma^2_{S'(AB')}$. This truncated $\mathcal{E}(\underline{MS})$ matches the $\mathcal{E}(\underline{MS})$ associated with the source $s\sigma^2_{AB'}$, which thus defines the proper error term.

Rule 5b.

To test any source of variation, put the obtained mean square associated with a source over the obtained mean square associated with the proper error term,

Table 6

The Generation of Expected Mean Squares

Illustrated with an $S'_{sabc}(A_aB_bC_c)$ Design

Sources	$\mathcal{E}$(MS) under H_1 (abbreviated notation)	F (df_{num}/df_{den})
$bcs\sigma^2_A$	A*+S(ABC)#	$MS_A/MS_{S'(ABC)}$@
$acs\sigma^2_B$	B+S'(ABC)	$MS_B/MS_{S'(ABC)}$
$abs\sigma^2_C$	C+S'(ABC)	$MS_C/MS_{S'(ABC)}$
$cs\sigma^2_{AB}$	AB+S'(ABC)	$MS_{AB}/MS_{S'(ABC)}$
$bs\sigma^2_{AC}$	AC+S'(ABC)	$MS_{AC}/MS_{S'(ABC)}$
$as\sigma^2_{BC}$	BC+S'(ABC)	$MS_{BC}/MS_{S'(ABC)}$
$s\sigma^2_{ABC}$	ABC+S'(ABC)	$MS_{ABC}/MS_{S'(ABC)}$
$\sigma^2_{S'(ABC)}$	S'(ABC)	

* Rule 4a. # Rules 4b through 4e. @ Rules 5a and 5b.

Table 7

The Generation of Expected Mean Squares

Illustrated with an $S'_{sabc}(AB'C)$ Design

Sources	$\mathcal{E}$(MS) under H_1	F
$bcs\sigma^2_A$	A*+AB'+S'(AB'C)#	$MS_A/MS_{AB'}$@
$acs\sigma^2_{B'}$	B'+S'(AB'C)	$MS_B/MS_{S'(AB'C)}$
$abs\sigma^2_C$	C+B'C+S'(AB'C)	$MS_C/MS_{B'C}$
$cs\sigma^2_{AB'}$	AB'+S'(AB'C)	$MS_{AB'}/MS_{S'(AB'C)}$
$bs\sigma^2_{AC}$	AC+AB'C+S'(AB'C)	$MS_{AC}/MS_{AB'C}$
$as\sigma^2_{B'C}$	B'C+S'(AB'C)	$MS_{B'C}/MS_{S'(AB'C)}$
$s\sigma^2_{AB'C}$	AB'C+S'(AB'C)	$MS_{AB'C}/MS_{S'(AB'C)}$
$\sigma^2_{S'(AB'C)}$	S'(AB'C)	

* Rule 4a. # Rules 4b through 4e. @ Rules 5a and 5b.

Table 8
The Generation of Expected Mean Squares
Illustrated with an $S'_{sabc}(A'BC')$ Design

Sources	$\mathcal{E}$(MS) under H_1	F
$bcs\sigma^2_{A'}$	A'*+A'C'+S'(A'BC')#	$MS_{A'}/MS_{A'C'}$@
$acs\sigma^2_{B}$	B+A'B+BC'+A'BC'+S'(A'BC')	¢
$abs\sigma^2_{C'}$	C'+A'C'+S'(A'BC')	$MS_{C'}/MS_{A'C'}$
$cs\sigma^2_{A'B}$	A'B+A'BC'+S'(A'BC')	$MS_{A'B}/MS_{A'BC'}$
$bs\sigma^2_{A'C'}$	A'C'+S'(A'BC')	$MS_{A'C'}/MS_{S'(A'BC')}$
$as\sigma^2_{BC'}$	BC'+A'BC'+S'(A'BC')	$MS_{BC'}/MS_{A'BC'}$
$s\sigma^2_{A'BC'}$	A'BC'+S'(A'BC')	$MS_{A'BC'}/MS_{S'(A'BC')}$
$\sigma^2_{S'(A'BC')}$	S'(A'BC')	

* Rule 4a. # Rules 4b through 4e. @ Rules 5a and 5b.
¢ A valid F ratio does not exist. An approximate F ratio may be derived using linear combinations of expected mean squares.

Table 9
The Generation of Expected Mean Squares
Illustrated with an $S'_{sabc}(A'B'C')$ Design

Sources	$\mathcal{E}$(MS) under H_1	F
$bcs\sigma^2_{A'}$	A'*+A'B'+A'C'+A'B'C'+S'(A'B'C')#	¢
$acs\sigma^2_{B'}$	B'+A'B'+B'C'+A'B'C'+S'(A'B'C')	¢
$abs\sigma^2_{C'}$	C'+A'C'+B'C'+A'B'C'+S'(A'B'C')	¢
$cs\sigma^2_{A'B'}$	A'B'+A'B'C'+S'(A'B'C')	$MS_{A'B'}/MS_{A'B'C'}$@
$bs\sigma^2_{A'C'}$	A'C'+A'B'C'+S'(A'B'C')	$MS_{A'C'}/MS_{A'B'C'}$
$as\sigma^2_{B'C'}$	B'C'+A'B'C'+S'(A'B'C')	$MS_{B'C'}/MS_{A'B'C'}$
$s\sigma^2_{A'B'C'}$	A'B'C'+S'(A'B'C')	$MS_{A'B'C'}/MS_{S'(A'B'C')}$
$\sigma^2_{S'(A'B'C')}$	S'(A'B'C')	

* Rule 4a. # Rules 4b through 4e. @ Rules 5a and 5b.
¢ A valid F ratio does not exist. An approximate F ratio may be derived using linear combinations of expected mean squares.

that is,

$$\underline{F}(\alpha) = \frac{\underline{SS}(\alpha) \ / \ \underline{df}(\alpha)}{\underline{SS}(\alpha_\epsilon)/ \ \underline{df}(\alpha_\epsilon)} = \frac{\underline{MS}(\alpha)}{\underline{MS}(\alpha_\epsilon)},$$

where $\underline{F}$ has $\underline{df}(\alpha)$ and $\underline{df}(\alpha_\epsilon)$ degrees of freedom.

Illustration: The $\underline{F}$ ratio for the source described above is $\underline{MS}(bs\sigma^2_A)/\underline{MS}(s\sigma^2_{AB'})$ or, abbreviated, $\underline{F}_A = \underline{MS}_A/\underline{MS}_{AB'}$.

Appendix

The sums of squares, algebraic and computational, and the degrees of freedom, associated with all sources of variation of various elementary designs are presented in complete form in the tables of the appendix.

Appendix - 1: Analysis of Variance (one-factor arrangement)

S'_sA_a repeated-measure design

Source	Σx^2: Algebraic (SS)	df	Σx^2: Computational (SS)
$s\sigma^2_A$	$s\sum^{a}(\bar{A}_a-\bar{M})^2$	a-1	$\frac{\sum^{a}(\sum^{s}X)^2}{s} - \frac{(\sum^{a}\sum^{s}X)^2}{as}$
$a\sigma^2_{S'}$	$a\sum^{s}(\bar{S}_s-\bar{M})^2$	s-1	$\frac{\sum^{s}(\sum^{a}X)^2}{a} - \frac{(\sum^{a}\sum^{s}X)^2}{as}$
$\sigma^2_{AS'}$	$\sum^{a}\sum^{s}(\overline{AS}_{as}-\bar{A}_a-\bar{S}_s+\bar{M})^2$	as-a-s+1	$\sum^{a}\sum^{s}X^2 - \frac{\sum^{a}(\sum^{s}X)^2}{s} - \frac{\sum^{s}(\sum^{a}X)^2}{a} + \frac{(\sum^{a}\sum^{s}X)^2}{as}$
σ^2_{total}	$\sum^{a}\sum^{s}(\overline{AS}_{as}-\bar{M})^2$	as-1	$\sum^{a}\sum^{s}X^2 - \frac{(\sum^{a}\sum^{s}X)^2}{as}$

$S'_{as}(A_a)$ independent-measure design

Source	Σx^2: Algebraic (SS)	df	Σx^2: Computational (SS)
$s\sigma^2_A$	$s\sum^{a}(\bar{A}_a-\bar{M})^2$	a-1	$\frac{\sum^{a}(\sum^{s}X)^2}{s} - \frac{(\sum^{a}\sum^{s}X)^2}{as}$
$\sigma^2_{S'(A)}$	$\sum^{a}\sum^{s}(\overline{AS}_{as}-\bar{A}_a)^2$	as-a	$\sum^{a}\sum^{s}X^2 - \frac{\sum^{a}(\sum^{s}X)^2}{s}$
σ^2_{total}	$\sum^{a}\sum^{s}(\overline{AS}_{as}-\bar{M})^2$	as-1	$\sum^{a}\sum^{s}X^2 - \frac{(\sum^{a}\sum^{s}X)^2}{as}$

Note: $\overline{AS}_{as} = X_{as}$

Appendix - 2: $S'_s A_a B_b$ Analysis of Variance

pure repeated-measure design

Source	Σx^2: Algebraic (SS)	df	Σx^2 : Computational (SS)
$bs\sigma^2_A$	$bs\sum^{a}(\overline{A}_a-\overline{M})^2$	a−1	$\frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$as\sigma^2_B$	$as\sum^{b}(\overline{B}_b-\overline{M})^2$	b−1	$\frac{\sum^{b}(\sum^{a}\sum^{s}X)^2}{as} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$ab\sigma^2_{S'}$	$ab\sum^{s}(\overline{S}_s-\overline{M})^2$	s−1	$\frac{\sum^{s}(\sum^{a}\sum^{b}X)^2}{ab} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$s\sigma^2_{AB}$	$s\sum^{a}\sum^{b}(\overline{AB}_{ab}-\overline{A}_a-\overline{B}_b+\overline{M})^2$	ab−a−b+1	$\frac{\sum^{a}\sum^{b}(\sum^{s}X)^2}{s} - \frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} - \frac{\sum^{b}(\sum^{a}\sum^{s}X)^2}{as} + \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$b\sigma^2_{AS'}$	$b\sum^{a}\sum^{s}(\overline{AS}_{as}-\overline{A}_a-\overline{S}_s+\overline{M})^2$	as−a−s+1	$\frac{\sum^{a}\sum^{s}(\sum^{b}X)^2}{b} - \frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} - \frac{\sum^{s}(\sum^{a}\sum^{b}X)^2}{ab} + \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$a\sigma^2_{BS'}$	$a\sum^{b}\sum^{s}(\overline{BS}_{bs}-\overline{B}_b-\overline{S}_s+\overline{M})^2$	bs−b−s+1	$\frac{\sum^{b}\sum^{s}(\sum^{a}X)^2}{a} - \frac{\sum^{b}(\sum^{a}\sum^{s}X)^2}{as} - \frac{\sum^{s}(\sum^{a}\sum^{b}X)^2}{ab} + \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$\sigma^2_{ABS'}$	$\sum^{a}\sum^{b}\sum^{s}(\overline{ABS}_{abs}-\overline{AB}_{ab}-\overline{AS}_{as}-\overline{BS}_{bs}$ $+\overline{A}_a+\overline{B}_b+\overline{S}_s-\overline{M})^2$	abs−ab−as−bs +a+b+s−1	$\sum^{a}\sum^{b}\sum^{s}X^2 - \frac{\sum^{a}\sum^{b}(\sum^{s}X)^2}{s} - \frac{\sum^{a}\sum^{s}(\sum^{b}X)^2}{b} - \frac{\sum^{b}\sum^{s}(\sum^{a}X)^2}{a}$ $+ \frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} + \frac{\sum^{b}(\sum^{a}\sum^{s}X)^2}{as} + \frac{\sum^{s}(\sum^{a}\sum^{b}X)^2}{ab} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
σ^2_{total}	$\sum^{a}\sum^{b}\sum^{s}(X_{abs}-\overline{M})^2$	abs−1	$\sum^{a}\sum^{b}\sum^{s}X^2 - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$

Note: $\overline{ABS}_{abs} = X_{abs}$

Appendix - 3: $S'_{as}(A_a)B_b$ Analysis of Variance

mixed design

Source	Σx^2: Algebraic (SS)	df	Σx^2 : Computational (SS)
$bs\sigma^2_A$	$bs\sum^{a}(\bar{A}_a-\bar{M})^2$	a-1	$\frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$as\sigma^2_B$	$as\sum^{b}(\bar{B}_b-\bar{M})^2$	b-1	$\frac{\sum^{b}(\sum^{a}\sum^{s}X)^2}{as} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$s\sigma^2_{AB}$	$s\sum^{a}\sum^{b}(\overline{AB}_{ab}-\bar{A}_a-\bar{B}_b+\bar{M})^2$	ab-a-b+1	$\frac{\sum^{a}\sum^{b}(\sum^{s}X)^2}{s} - \frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} - \frac{\sum^{b}(\sum^{a}\sum^{s}X)}{as} + \frac{(\sum^{a}\sum^{b}\sum^{s}X)}{abs}$
$b\sigma^2_{S'(A)}$	$b\sum^{a}\sum^{s}(\overline{AS}_{as}-\bar{A}_a)^2$	as-a	$\frac{\sum^{a}\sum^{s}(\sum^{b}X)^2}{b} - \frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs}$
$\sigma^2_{S'(A)B}$	$\sum^{a}\sum^{b}\sum^{s}(\overline{ABS}_{abs}-\overline{AB}_{ab}-\overline{AS}_{as}+\bar{A}_a)^2$	abs-ab-as+a	$\sum^{a}\sum^{b}\sum^{s}X^2 - \frac{\sum^{a}\sum^{b}(\sum^{s}X)^2}{s} - \frac{\sum^{a}\sum^{s}(\sum^{b}X)^2}{b} + \frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs}$
σ^2_{total}	$\sum^{a}\sum^{b}\sum^{s}(X_{abs}-\bar{M})^2$	abs-1	$\sum^{a}\sum^{b}\sum^{s}X^2 - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$

Note: $\overline{ABS}_{abs} = X_{abs}$

Appendix - 4: $S'_{abs}(A_a B_b)$ Analysis of Variance

pure independent-measure design

Source	Σx^2: Algebraic (<u>SS</u>)	<u>df</u>	Σx^2 : Computational (<u>SS</u>)
$bs\sigma^2_A$	$bs\sum^{a}(\bar{A}_a-\bar{M})^2$	a−1	$\frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$as\sigma^2_B$	$as\sum^{b}(\bar{B}_b-\bar{M})^2$	b−1	$\frac{\sum^{b}(\sum^{a}\sum^{s}X)^2}{as} - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$s\sigma^2_{AB}$	$s\sum^{a}\sum^{b}(\overline{AB}_{ab}-\bar{A}_a-\bar{B}_b+\bar{M})^2$	ab−a−b+1	$\frac{\sum^{a}\sum^{b}(\sum^{s}X)^2}{s} - \frac{\sum^{a}(\sum^{b}\sum^{s}X)^2}{bs} - \frac{\sum^{b}(\sum^{a}\sum^{s}X)^2}{as} + \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$
$\sigma^2_{S'(AB)}$	$\sum^{a}\sum^{b}\sum^{s}(\overline{ABS}_{abs}-\overline{AB}_{ab})^2$	abs−ab	$\sum^{a}\sum^{b}\sum^{s}X^2 - \frac{\sum^{a}\sum^{b}(\sum^{s}X)^2}{s}$
σ^2_{total}	$\sum^{a}\sum^{b}\sum^{s}(X_{abs}-\bar{M})^2$	abs−1	$\sum^{a}\sum^{b}\sum^{s}X^2 - \frac{(\sum^{a}\sum^{b}\sum^{s}X)^2}{abs}$

Note: $\overline{ABS}_{abs} = X_{abs}$

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