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Geometry and the Cognitive Principle in Semiotics and Esthetics

Dedicated to Dietrich Mahlow on the occasion of his 75th birthday

1 Introduction

Preliminary considerations to follow give some introduction to the other chapters of this paper, where specific remarks and references to literature will also be found.

One encounters geometric objects everywhere. Plane or solid forms can be discovered in plants and animals, in pieces of art, machines and designs. The geometric domain not only comprises simple forms like circle and triangle, but complicated surfaces and bodies as well. Recently, the fractals have joined the great family of geometric patterns, irregular elements, which become evident in the ramifications of lightnings and rivers, likewise in the contour of mountains or even the bark of trees. Nowadays, of course, the majority of geometric forms will reach the beholder from the screens of television sets or computers.

When drawing geometric figures and diagrams, one creates visible and manipulatable signs which refer to those ideal geometric objects which are subjected to the rules of mathematics, logic and calculus. Now, rule-conducted discussion of signs is the concern of the science of semiotics. However, any earnest investigation of geometric signs must transcend even semiotics, because geometry, computation, finally semiotics itself are in numerous ways linked to the grand realm of the esthetic state. This will be fully perceived by anyone who has endeavoured to relate mentally e.g. the abstract circle to artfully constructed wheels and their dynamic laws, or simply to the graceful affinity of the sunflower to the sun.

Though as soon as we take the side of computation and logical deduction, we have joined already a methodology which Leibniz preconceived as his 'characteristica universalis' which is, however, now generally known as 'cognitive'. Consequentially we will in this paper attempt to apply cognitive procedures to pattern-oriented semiotics and esthetics. This question is posing our central problem: "How can beautiful forms and signs be controlled by the calculus?" If we will be as lucky as to find useful answers, we should already be deeply involved with the foundation of 'cognitive' modes both of semiotics and esthetics.

With chapter 2, we start the exploration of cognitive tools in semiotics and esthetics. Our investigations are based on the semiotic theory, established by

Charles Sanders Peirce, carried on by Max Bense and applied by this 20th century philosopher to numerous modern problems. According to Peirce's concept, any complete sign is given by a triadic relation S = (M O I) between three elements: Whereas M refers to the medium of the sign S, O refers to the object which is denoted by S; finally I refers to the interpretant by which the meaning of S is given. Triadic signs can be affected by modulation when subjected to the processes of semiosis. Indeed, cognition itself for which the signs are serving, can be interpreted as the all-encompassing semiosis (2.1).

The variety of signs allows a subdivision into ten main classes which reach from the 'iconic-rhematic-qualisign' to the 'symbolic-argumentic legisign'. In order to tie this classification to our main subject, namely geometry, we will trace semiotic features of the following two sign samples through all ten main classes: 1. The 'Black Square' of the artist K. Malevitch; 2. the square considered in a plain geometric sense (2.2).

Max Bense's deep interest in the field of semiotics is particularly displayed by his work on the foundation of rational esthetics. Cautious analysis can reveal three phases in his profound engagement with beauty. The first is familiar from early books by the philosopher, the second from his main work 'Aesthetica'. During a third phase, Bense developed esthetics as the theory of 'self-reality'. This last phase is the most important to our concern, because it leads to a conclusive definition of the geometric-esthetic sign (2.3).

Chapter 3 is dedicated to perception as being mediated by signs. Here, valuation and enjoyment of beauty as self-reality is already announcing itself. Symmetry proves to be the core idea in geometry (3.1). The concept of the 'geometric-esthetic nexus (genex)' is introduced by which we understand the diversity or 'semiotope' of all esthetically efficacious signs. According to a metaphoric interpretation, the semiotope of the genex is an evolving system in which signs cooperate and concur. We shall speak of the 'sign-game' as intrinsic to the genex, the game in which the 'man-sign' is one only among a multitude. (3.2).

With regard to semiosis in the genex, one particular epistemological standard behavior of the geometry-promoting man-sign is prominent. This behavior which we call the 'geometric-semiotic bypass', shows itself in automatic and unaware transitions from geometric diagrams to the mathematical forms and facts intended by those diagrams. For instance, a circle scratched in the mud with a stick will nevertheless normally intend the ideal circle. We understand the bypass as a general economical principle of practising geometry. One most important feature of the bypass is the running and controlled observation of geometric laws, this observation guaranteeing the logical consistency of the geometric process as a whole (3.3).

In order to comprise fractals and chaos among the elements of the genex, we introduce the notion of granularity. Not only are geometric objects granulated differently, rather will the scale of granulation itself constitute a fractal and a geometric principle at the same time (3.4).

Chapter 4 finally presents the formal and computational apparatus through which cognitive semiotics and esthetics become possible. We introduce the 'cognitive description' d, a mathematical function that maps a sign S = (M O I) onto its 'descriptor sign' dS = (dM dO dI). The sign dS proves to comply to the syntax of the idiom LISP, the outstanding programming language in the field of Artificial Intelligence. This procedure unlocks the methodological stock of LISP and Artificial Intelligence to semiotics; in other words: We can begin to compute. Certainly the mapping d needs to be a functor in the sense of the mathematical theory of categories. This theory, however, was introduced by Bense already into his construction of semiotics (4.1).

When operating within the framework of cognitive sign description, signs must be considered to be able to act as agile agents in cognitive sign-games. To be sure in a scenario like this, only living agents, especially human ones functioning as man-signs, can have real sensations. On the other hand, pseudo-sensations may be formally adjudged to machine-like agents (4.2). When brains are described cognitively, the incompatibility of sensations and pseudo-sensations has one important consequence: Consciousness cannot be in the brain (4.3).

Chapter 5, ultimately, returns to the topic of the genex. We are going to formulate the concept of cognitive esthetics. Following Bense, the 'stripping' of a sign from all its extra-esthetic references can be conceived as a forgetful functor. We call this functor, which produces self-real signs, Bense's esthetificator. Indeed cognitive-descriptive methodology makes possible the construction of even more flexible esthetificators which are able to store the 'semantical corona' of a sign in the descriptor dI of its interpretant (5.1).

Then, we apply the cognitive method to the field of geometry-oriented constructive-concrete art. If, according to Bense, the mediation of any piece of art is realized to be a process of communication, then one expedient-bound as well as one recipient-bound repertory of any stream of signs to be communicated must be taken into account. Citing Dietrich Mahlow, every beholder sees the transmitted signal in the mode of his personal 'mental image'. Besides that, now an 'inverse bypass' must be in action, which keeps ideal geometric objects rather in the background. One can conclude that mental pictures are beholder-local interpretations of 'geometric filial signs' of ideal background objects (5.2).

In this age, 'computational geometry' and 'Computer Aided Design (CAD)' release wide streams of geometric signs, becoming evident in the internal structures and on the external hulls of myriads of artifacts. Hence, we are presented with calculatory generated facts not explicitely addressed, even if clearly included, in our own cognitive analysis of the sign scene (5.3). Recently, sign domains called 'virtual worlds' are entering the genex, which generate complete geometric universes. To be sure, the method of cognitive description allows the analysis of these innovative computational realities (5.4).

Apparently, the evolution of the operative engineering man is approaching a climax which in the best case should discharge into catharsis. This is because the mechanical constituents of 'his' genex are displaying the tendency to grow into an entangled jungle of proliferating contraptions. A major question ought to be put: Can one think of a universal goal for research which is closely bound up with geometry, yet would at the same time be able to focus all efforts for a hygiene of the sign? As a matter of fact, the ancient dream of artificial man permits cognitive interpretations which could bring together many intellects around one single table. Then, the unmarred geometry and symmetry of the human body and organization could find its radical innovative counterpart, to be seen like in a mirror (5.5).

2 Charles Sanders Peirce, Max Bense, the cognitive factor

2.1 Firstness, secondness, thirdness

Letters of Ch. S. Peirce to Lady Welby contain a concise draft of the thinker's sign theory [1]. Here, he drew up a list of the notions which much later, yet unaltered, also served as a fundament of Max Bense's independent architecture of the sign cosmos [2]. Following Peirce, the three basic categories are 'firstness': the mode of that what is as it is, without relations to something else; then 'secondness': the mode of that which is related to a second, without consideration of any third; finally 'thirdness': the mode of that which relates a second to a third.

In the progress of his work, Peirce provided for an abundance of interpretations of his basic categories. Thus, the following explication is fundamental to Peirce's semiotics: The 'second' appearing in secondness can be considered as a non-ego as opposed to an ego, hence as an 'object'. However, an interpreting meaningful thought or 'interpretant' has to relate a firstness to its object. Now, the complete definition of the sign becomes possible: It is mediating between the interpretant and the object meant. This definition allows immediately the formulation of the sign as a 'triadic' relation:

(1) S^3 = (medium object interpretant)

Bense used to handle this relation with the following particular abbreviation:

(2) S^3 = (M O I)

Any complete sign in Peirce-Bense semiotics is distinguished by the 'trichotomic' structure (1) or else (2) [3].

Generally, logic deduction and calculation rank very high in Peirce's philosophy. We will in chapter 4 absolutely comply to this characteristic feature by including symbolic operations to our cognitive approach. This will basically be accomplished by mapping S^3 to a list-like formal expression

(3) dS = (dM dO dI)

of the programming language LISP, intrinsic to Artificial Intelligence [4]. To be sure, the expression dS will then prove to be a genuine complete sign.

2.2 The ten main classes of signs

In order to obtain a triadic 'micro-classification' of all signs, Peirce makes a threefold use of each of the basic categories firstness, secondness, thirdness [5]. Proceeding thus, he lets himself be guided by the three references of any sign, namely respectively to its medium, its object and its interpretant. Part of this procedure is the supplementation of class-names by a numerical prefix 1 to 3. The numerical notation applied by us is, by the way, deviating by small differences from the 'official' one used in papers by Bense.

Then, following Peirce again, we learn that with respect to medium-reference the '1-qualisign' can be understood as a phenomenon, the 2-sinsign' as an individual object or event, finally the '3-legisign' as something legible. Likewise, we realize that with respect to 'object-reference' the '1-iconic sign' is determined by its inner nature, the '2-indexicalic sign' by its intrinsic relation to its object; finally the '3-symbolic sign is distinguished by its detailed determination by an interpretant. The third and final micro-classification is supplied by the interpretant-reference: The '1-rhematic sign' can by its quality as a proper or class noun be given a name, the '2-dicentic sign' can be asserted or denied, the '3-argumentic sign' even proved or refuted.

By augmenting the numerical characterization of signs in a straightforward way, micro-classification will yield sign-classes from '111-rhematic-iconic qualisigns' up to '333-argumentic-symbolic legisigns'. There is, however, a severe constraint on the generation of proper 'main classes'. This constraint is given by the numerically definable condition of 'ascending semioticity'. In order to illustrate this important principle, we choose as an example a rather famous sign, namely the painting known as the 'Black Square' by the artist K. Malevitch. From this 'master sign' we will derive ten different signs, each belonging to a main class, each granting a particular scope of Malevitch's masterwork. To be sure, each of these ten visual signs is in the lines to follow appearing only as presented by a class-characterization, augmented by an interpretative text:

111	(Qua Ic Rh)	A black square.
211	(Sin Ic Rh)	This black square.
221	(Sin In Rh)	The 'Black Square' of Malevitch.
222	(Sin In Di)	The 'Black Square' of Malevitch, considered as outstanding in art history.
311	(Leg Ic Rh)	Black square beheld as a black square.
321	(Leg In Rh)	The 'Black Square' of Malevitch, beheld as such.
322	(Leg In Di)	The 'Black Square' of Malevitch, to be read as outstanding in art history.
331	(Leg Sy Rh)	Black square as symbol of scarcity.
332	(Leg Sy Di)	Black Square as symbol for: "Concrete art is to be encouraged!"
333	(Leg Sy Ar)	Black square as symbol for: "If funds are available, concrete art can be promoted."

One sign class is absolutely eminent among the ten main classes: 321 which Bense declared to be the domain of all auto-referring hence 'self-real' signs. He even identified this domain with esthetics as such, as well as with the quality of being a sign and a number [6]. With respect to our example, the 'Black Square' of Malevitch, beheld as such, is not only acting as a typical 321-sign, but it is our 'master sign' we have now recurred to, hence sort of a 'germ' of its ten derivations.

Already from the example of the black square, the relevance follows of a proper classification of signs to esthetic and geometric questions. With our next example, visual signs are not only mentioned but shown. Thus figure 1 displays ten geometric signs, interpreted as follows in the style we are now familiar with:

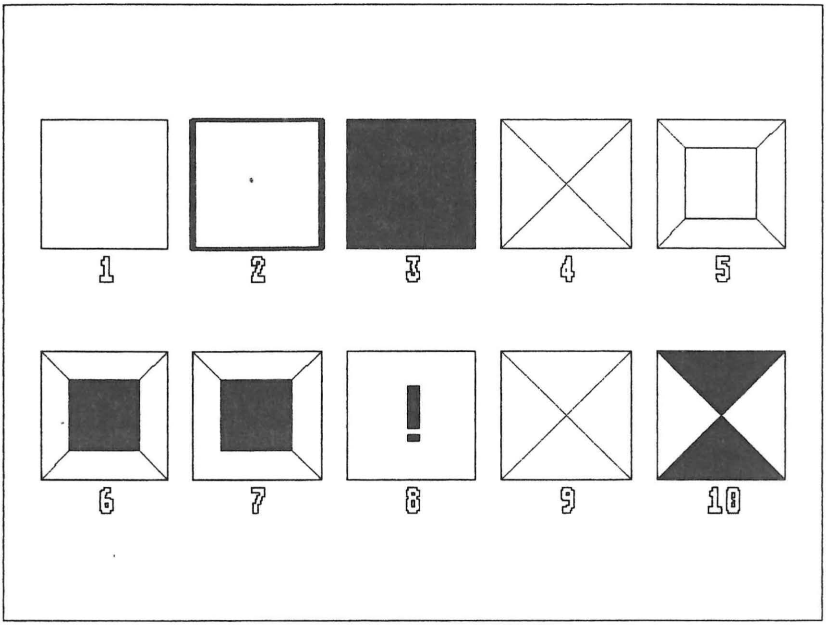

Figure 1. Ten geometric signs

111	(Qua Ic Rh)	Square.
211	(Sin Ic Rh)	Square particularly outlined.
221	(Sin In Rh)	This black square.
222	(Sin In Di)	This black square, showing diagonals of equal length.
311	(Leg Ic Rh)	Square confirmed as stable inside a square.
321	(Leg In Rh)	Square auto-referring ornate.
322	(Leg In Di)	This square showing its flaw.
331	(Leg Sy Rh)	Symbol of the science of geometry.
332	(Leg Sy Di)	"I square have diagonals of equal length."
333	(Leg Sy Ar)	"If I am a square then I am enclosing four rectangular and congruent triangles."

One will note at once that each of these interpretations of one of the ten diagrams in figure 1 is neither compelling nor unique. This, of course, is a result of the contingent liberty of the sign-creating authority, say interpretant. Although any interpretation will surely depend on a preexistent environment of the interpretant which Bense called the 'Interpretantenfeld' [7] and which we will later discuss as relevant to the 'semantic corona' of the sign (5.1).

2.3 Max Bense's theory on beauty

Considering Bense's attitude towards esthetics, three consecutive phases can be discerned [8]. The first of these may be grasped by reading his early works, especially the books "Geist der Mathematik" and "Die Mathematik in der Kunst" [9]. Here, the former one already contains a chapter "Mathematik und Ästhetik". In the other book the following sentence is found: "Among mathematical forms there is a special class of forms of the geometric and arithmetic kind which is distinctive by affecting our senses, i.e. they not only can be thought but perceived, they also are not only perceived, but are touching, transforming us, and these 'affecting' forms are called by us esthetic forms" [10].

A culmination of the second phase is marked in the year 1965 by the publication of the 'Aesthetica' [11]. During this period, Bense shaped the tool 'sign' into his main instrument for analyzing beauty. Already in part 4 and 5 of the 'Aesthetica', the sign theory by Peirce is used in a constructive way. Bense coined the notion 'esthetic state': "Conversion of material extensional 'elements' (sounds, colors, strokes, words) into 'signs' is the first decisive incident by which a physical state is transformed into an esthetic one" [12]. According to Bense, the esthetic state of any piece of art is fragile and of low probability.

At that time, Bense added two topical concepts to his repertory of thinking: 'Information' and 'program'. In the years to follow, he again and again touched the rôle of the computer in esthetics [13]. He also engaged himself in the development of the field of numerical information-esthetics. In his book "Einführung in die informationstheoretische Ästhetik" he draws attention to considerations of A. A. Moles and himself, which later led R. Gunzenhäuser to the conception of a measuring theory for unorder and complexity [14].

Bense succeeded in defining the intrinsic nexus of the conceptual triple esthetic information, style, redundancy. When in the esthetic state, the object is communicating esthetic information. Bense not only distinguishes esthetic redundancy as a mode of order, but determines its essence as repetition of the equal, the known, the predictable [15]. Here, redundancy proves to be the subtractive term in esthetic information, because it is by repetition that redundancy is exactly counteracting to the singularity of works of art. But from hence, the phenomenon of style comes into existence [16].

In an extensive final phase of his lifelong work in the field of esthetics, Bense not only solved an abundance of mutually linked problems, but also approached his own unique cosmologic-esthetic vision. This phase is disclosed to us by not less than eight books from the years 1967 to 1986 [17], followed by "Die Eigenrealität der

Zeichen", published in 1992 from the literary remains by Elisabeth Walther. The title of this book again names the notion, perfectly characterizing Bense's reflection on the signs and the world: 'Eigenrealität', i.e. the 'self-reality' of the esthetic state, of the number, finally of the sign itself [18].

3 The geometric-esthetic nexus (genex)

3.1 Sign, form, symmetry and self-reality

Man, as we know him, deals with fire, speaks and manifactures artifacts. When we consider the structure of cooperation between perception and action, between eye and hand, then nothing indicates fundamental differences between ice-age man and contemporary man. In their ways of proceeding and succeding, human beings distinguish the individual from the universal, the concrete from the abstract: "He killed the deer by his spear", "This computer program was finished by her during the last half-hour", "Deer tastes good", "AI-programming is a ticklish thing", "Your deer on the back cave wall just lives", "The colored squares in this picture are delicately counterbalanced." At all times, signs were particularly then used, when the percepted had to be mastered by work and speech.

That which comes upon us or is produced by man is differentiated by him by its quality: "This is better, more useful, of higher value, nicer than that." Sometimes the valuation is accompanied by an experience of perfection: "The chieftain's speech was faultless", "K's representation of the sphere is the sphere." In this case, the thing or event valuated may stand out and at the same time be pointing i.e. referring to itself only. Considered as a semiotic process, here a sign is met cutting away its cables tied to any external meanings, and showing its individuality unveiled. It crosses over to self-reality [19] and moves into the esthetic state.

Clearly, our topic is the connection between symmetry, regularity and form. Let us contemplate the following situation: Someome is turning around in his hands a sphere; may be it is a bearing-ball, smooth as a mirror, without any bumps. Then he stops short and puts these questions: "Is this really a sphere? Is it correct plainly to say of the sphere that it is able to reflect my face?" Then he proceeds: "Surely I am not going to be reprimanded by anyone when I say not more about this spherical ball, as that any of its spots is indistinguishable from every other one [20]. On the other hand, I may confirm rightfully that this bearing-ball is sphere-shaped, it is of the form of the sphere." Now we understand that the word-symbols or special signs 'form' and 'sphere' are denoting universals.

Moreover, we are realizing that the esthetic state as such is distingished by regularities to which at any case do belong the great paradigms of symmetry, i.e the bilateral proportion of the gestalt of man, or else the rotational symmetry of the sphere. On the other hand, regularity is as well revealed by redundant repetition of aleatorically varied features of any shape. For instance, the general building plan of sea shells is recognized easily, although they are found in countless variants of form and color. In fact, the term 'redundancy' as understood by Max Bense (3.4), means nothing else than regularity in the sense of generalized symmetry [21]. Thus the use of 'symmetry' as synonymous to 'redundancy' must be considered to be correct.

3.2 Semiosis and its metaphors

Signs are tools of man. However, at least the same weight has the realization that signs control the behavior of man. Thus, signs are of the essence of 'agens' as well. When we are reading or watching television, signs operate as the determining agents of our experience. At any time those signs check what we will perceive, think and feel in the moments just to follow. Therefore, it is clearly in the interest of intellectual sincerity to specify the metaphor of the sign as an agent more precisely. Thus we define: An agent is a control-sign distinguished by its typical behavior [22].

When we are using the metaphor of the agent in a way compatible with the metaphor of life, then agents must behave like individuals of varieties living together. This will include evolution. Peirce linked his own concept of evolution to his concept of categorical trichotomy: "Three modes of evolution have thus brought before us: evolution by fortuitous variation, evolution by mechanical necessity, and evolution by creative love" [23]. However, if man as an agent cooperates with the signs, then it is certainly consequential when he himself is considered to be of the character of signs. As a matter of fact, Peirce understands man as the man-sign [24].

Thus, signs influence each other like agents; they live together. But then, just another metaphor proves to be useful: Let us consider the all-comprising semiosis, i.e. the life of the signs, as a game [25]. At the same time we assume that the signs as the agents of semiosis create places where their own game can unfold. Those places are called by us semiotopes.

The human hand is acting as the counter-form to those tools, handled by it. This is the reason why phylogenetically early pattern-shaping engravings in stone or ivory, as well as ontogenetically early pencil drawings by children, point to the co-evolution of hand, tool, eye and picture [26]. From now on, we will use a special term for the characterization of the bond between geometricity and beauty: The semiotope of all signs which can be interpreted only by reference not just to geometry but also to esthetic self-reality, will be called the 'geometric-esthetic nexus'. In most cases we will indeed use the abbreviation 'genex'. The semiotic relevance of the genex is supported by two arguments which are tied up to the memory-functions and to the self-organization of signs:

1. If pictorial signs are materially stable, then they often are met travelling through time rather undisturbed. That simply means that they are functioning as storages for information. Signs of this kind mediate between man-signs and other agents in diverse phases of their respective existence. Without this memory-function of signs e.g. the phenomenon of art history is unthinkable.

This statement will aid to localize germs and vertices of the genex. Can one think of any other universal cause for growth processes intrinsic to the genex? Here again, we may resort to the concept of self-organization which, at present, is holding a center place in scientific discussion anyway[27 28]. The definition of self-organization used by us is compatible to one given by Uwe an der Heiden [29]. We combine the following argument with a principle of optimization:

2. We describe self-organization to the genesis of many pictorial signs in the following sense: In the course of the semiotic game, signs organize themselves in such a way that some of them, which often are parts of other signs, are seeking likeness to elementary geometric forms by adaptation of their own shape. Semiosis acts here as the local organizator. As a rule, manifestation of self-reality of signs will coincide with the culmination of their self-organization.

Pointlike marks and lines, to be engraved as optic as well as haptic signs, must be considered to be the simplest geometric forms. Therefore, elementarity of form must be ascribed also to arclike curves observable on artifacts and pictures from the ice-age, the perfectness of which has always been striking. To be sure, the appearance of geometric forms as components of the organization of artifacts will have been amplified by the use of tools. For instance, the leading of scrapers along rectilinear paths is clearly corresponding to the straight line as such. Throwing of the spear, shooting with the bow is mapped onto curves. The potter's wheel and the turning-lathe are linked to the complex form of the rotational body [30].

3.3 The geometric bypass and the foundations of geometry

Viewed from a semiotic viewpoint, recognizing and acting are processes intrinsic to semiosis. Using a metaphor again, the operating man-sign is perpetually catching objects and drawing them to it, whereby signs are the handles with which those objects are grasped. The very event of grasping is usually happening unconsciously, of course. As an example, let us watch a hunting team, already on reply are canalizing streams of acoustic and gestic signs, organized in the form of a network. With this process, semiosis is permanently leaping or bypassing the gap between signs and the objects designated, between "deer" and deer. This will not deny the distance between sign and object, but rather emphasize it. Let us call the very event of leaping over the gap the 'semiotic bypass'.

Patterns, shapes and geometric objects constituting our main topic, we will at once come across the special case of the semiotic bypass which may be called the 'geometric-esthetic bypass'. Geometric practise is throughout depending on the utilization of geometric diagrams in designating its objects. Figure 2 shows an example which Platon made use of in his dialog 'Menon'. Sokrates, by demonstrating this diagram to a servant of Menon, succeeds in awaking the man to the insight that the square drawn as a rhombus doubles the area of the hatched one [31]. To be sure, this diagram is interpreted by the servant not as a pattern in the mud, but by help of the geometric-esthetic bypass as a symbol denoting a mathematical proof; we today would consider it to be an argumentic-symbolic legisign.

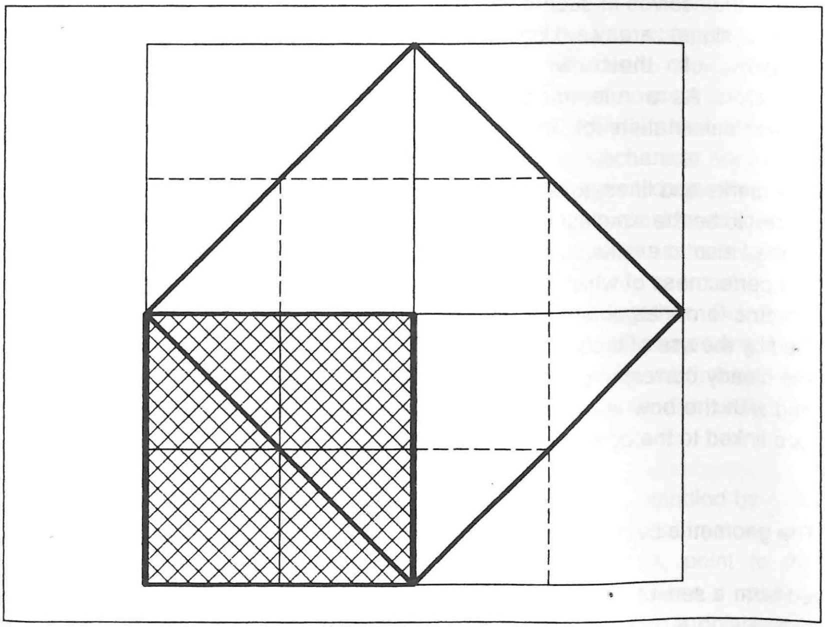

Figure 2. Menon's diagram

From time immemorial, geometric discussion has relied on the functioning of the geometric-esthetic bypass without further reflection. Obviously, geometric apprehension and action obey to an economic semiotic automatism. Later on, we will understand this process as a cognitive principle. At any time, the discourse is opened following this scheme [32]: "Given are points, lines, arcs and angles a, b, c; look for the diagram!" Therefore, it is the more remarkable, how clearly Platon recognized the snares laid here [33].

This is essential, however: The human sign-interpreter is not on principle compelled to perform the bypass. On the contrary, the constant option of conscious recurrence to the sign-medium belongs to the essence of the sign-interpretation by the man-sign. Thus, one could imagine Sokrates having drawn his diagram twice or thrice prior to his dialog, because it would have appeared to him as not suggestive enough. 'Not suggestive enough' does, of course, mean 'of insufficient precision to eye and mind', what is immediately to be considered as an esthetic quality.

Is the evident completeness of the ideal geometric object, constituting the target of the geometric-esthetic bypass, comprehensible by symbolic conceptualization? Again, we encounter symmetry. For instance, we say: "This pearl is spherical." However, we already know of the sphere form that it is passing into itself when turned around the center. This observation actually provides for a conceptualization of the symmetry of the sphere. Corresponding laws, however, hold for the rectangle, likewise for the square and every other regular polygone, when turned around their centers by the adequate amount of angle [34]. But can we possibly advance to still other principles of geometry which prove to be at least as fundamental as symmetry? Do perhaps evident statements concerning geometric objects exist, from which symmetries can then be derived? Indeed, there are such 'geometric axioms', even if existing not in a unique way. Though, as a matter of fact, only in this century a system of axioms was developed for Euclidean geometry which is safely excluding logical contradictions [35]. Thus, it has been the utilization of mathematical logic that is marking the beginning of a genuine geometric-mathematical science.

Impregnation of geometric thinking by logic has made possible the unfolding of theories which are renouncing largely the use of diagrams and other pictures. Many among those theories hold valid independent of any choice of a number of spatial dimension [36], even if there are severe limits to visualizing structures of a dimension higher than three. Figure 3 presents the vertices and edges of the cube of dimension five. By lines drawn more prominently, one of its numerous threedimensional subcubes was articulated.

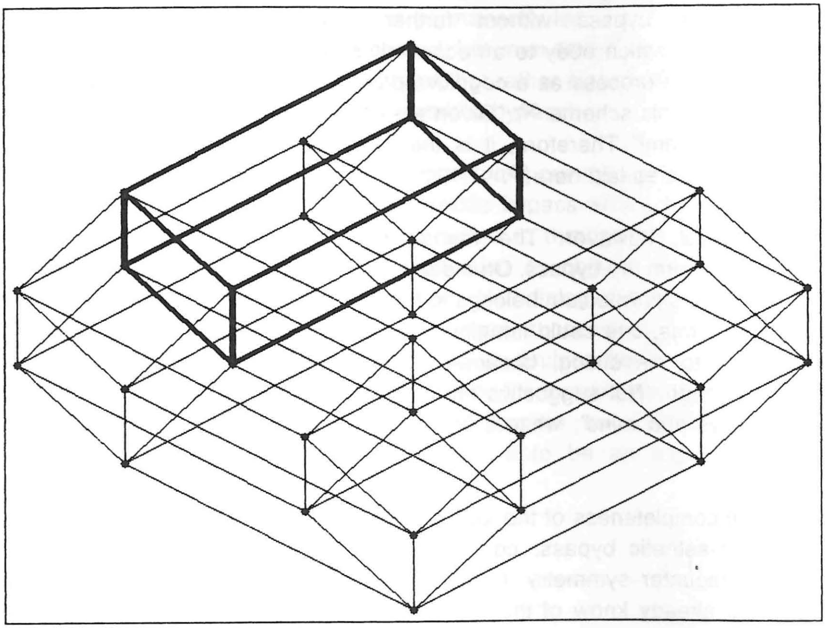

Figure 3. 5-dimensional cube with 3-dimensional subcube

What the unsensual semiotics of abstract geometric theories does in no way exclude is emotion. Thus, it is wellknown to mathematicians that the process of solving problems in the abstract realm can be experienced as a hunt. So here are the hunters again, although now the longed-for trophies are fascinating geometric objects as well as profound theorems. And there, the semiotic bypass is unceasingly effective. If, for instance, a theorem is ascertaining a symmetry, then, of course, not the statement as a word-symbolic dicentic text is the goal of the hunt, but rather the geometric relationships confirmed by the mathematical proof.

There is, however, an essential motivational difference between the artist and then the mathematician, because he, who is possibly not at all insensitive to the perfectness of a beautiful geometric diagram, will nevertheless be interested in it primarily as a helpful or even precious accessory. Hence, however, the 'Black Square' of Malevitch may be dear to the mathematician, yet not in his character as a professional. Therefore, an amalgamation or even fusion of geometry and art is absolutely unlikely, because it is excluded by deeply rooted semiotic reasons. As a consequence, growing mutual interest observable in the two professions must rather discover discourse as its form - which is not little, anyway.

3.4 Fractality, chaos, granularity

One will presumably agree that man does not yet well understand how his sign-bound visual perception is functioning at all [37]. However, one can also be certain that the stability of the structure of the appearing world must in the end be based on constants. To be sure, geometric objects together with their symmetries will have to be counted among those constant universals. Now, any symmetry is distinguished by its particular mode of repetition or redundancy. Yet, according to Bense, there are families of signs, the partial signs or sub-signs of which can be understood as selected from pre-established sign stocks, which Bense calls 'repertoires' [38]. Hence, the special arrangement of the partial signs of a sign may be seen as having come to pass by external determination by a selective construction piece by piece [39], however possibly also by conscious assemblation [40], or even by aleatoric processes of dispersion or growth [41].

It is in this context, where the signs and objects, discovered and named 'fractals' by B. Mandelbrot, turn out to be extremely meaningful [42]. Fractality, the particular mode of redundancy with a fractal, always is the repetition of basic shapes at any level of size. Mandelbrot mentions coastlines for an example, the arclike indentation of which reach from country-size bays down to intimate batheing coves. Fractality can be analyzed by mathematics, thereby disclosing a huge class of innovative geometric forms.

Closely related to fractals are the chaos orders [43]. A deterministic chaos in the sense of chaos-research is defined as a distribution of mathematical objects of an apparently probabilistic or irregular character which, nevertheless, can be computed by nonaleatoric i.e. deterministic rules [44]. Mandelbrot stated a postulate concerning the link between chaos and fractals, which says that a chaos will always occupate certain partial domains of a fractal [45]. As a matter of fact, if one is succeeding in the representation of a chaos order by a geometric diagram, then very often fractal form-repetitions of a stupendous variety will be revealed (figure 4). Moreover, there are chaos orders which display aleatoric enrichment (figure 5). If, for instance, a horizontal plane is mentally posited in such a way that it cuts the fractal root system of a tree, then the figure of all points of intersection with the most delicate roots will expose an aleatoric chaos order.

By the exploration of fractality and chaos, mathematics as well as physics and biology have demonstrated that the variety of geometric objects and their semiotic codifications is far more abundant than scientists of the nineteenth century could surely have guessed. In order to get some survey on the complete dominion of geometric signs, things and objects, we will now introduce the heuristic concept of granularity: Something is said to be the more subtly granulated or of the higher granularity, the richer its manifest intrinsic structure is. The square and the circle, or sections of smooth curves, are of a very low granularity according to this definition. At the other end of the graduation one can find, for instance, the geometric frameworks of nervous systems. Obviously, the granularity gamut of reality is traversing orders of ranking not easily thought of. To be sure, additional structure of the genex is induced by the so understood graduation of granularity. Granularity may as well provide for an esthetic principle of cosmologic order.

There is an important differentiation of all fractals into two classes. Fractals discovered in connection with scientific research, e.g. in physics or biology, are called by us 'analytic fractals'. On the other hand, there are the synthetic fractals, created in very conscious design processes in the esthetic laboratory. Chaos games, resulting in a specimen of the one or the other kind, are respectively called 'analytic' or 'synthetic' [46]. Overlapping of the two fractal classes may not only happen, but can also pose interesting problems. Figure 4 shows an analytic [47], figure 5 a synthetic fractal [48].

The relation of fractal geometric diagrams, i.e. graphic signs denoting fractals, to their respective fractal geometric objects, is in no way without problems. First, the structural echelon of form repetition inside most fractals is an infinite thing, a fact which may become evident simply by looking at figures 4 and 5. Then, the high granularity of fractals may evoke just the mixing up of diagram and object by the beholder who, forced to ponder the exact connections between both poles of semiosis, will likely be retarded in accomplishing the semiotic bypass. This again may result in positive effects, of course, because the beholder's analytic reactions may very well reveal innovative structural features of the diagram beheld and the object referred to. An even more dramatic situation will be taking place when the diagram enters self-reality. In this most significant case, the beholder stops short of the bypass anyway. Later on, we will distinguish this event as the inverse bypass, the existence of which has been the main reason for our dealing rather thoroughly with the topic of the bypass at all.

4 Cognitive semiotics

4.1 The functorial definition of cognition

What is the meaning of 'cognitive'? The adjective derives from the Latin noun 'cognitio', i.e. 'cognition', 'conception', or even more generally 'acquisition of knowledge' [49]. However, what we have in mind is an intensified usage of these terms which, when following Leibniz, prefers computational problem solving to any other methods [50]. D. Münch in a paper recognizes data processing as the methodical base of cognitive procedures [51], although this interpretation is too

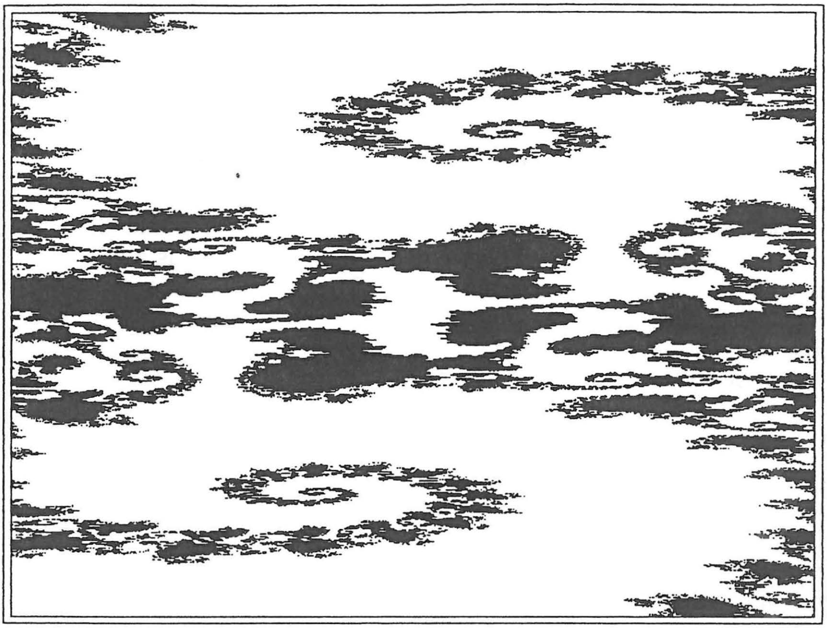

Figure 4. An analytic fractal

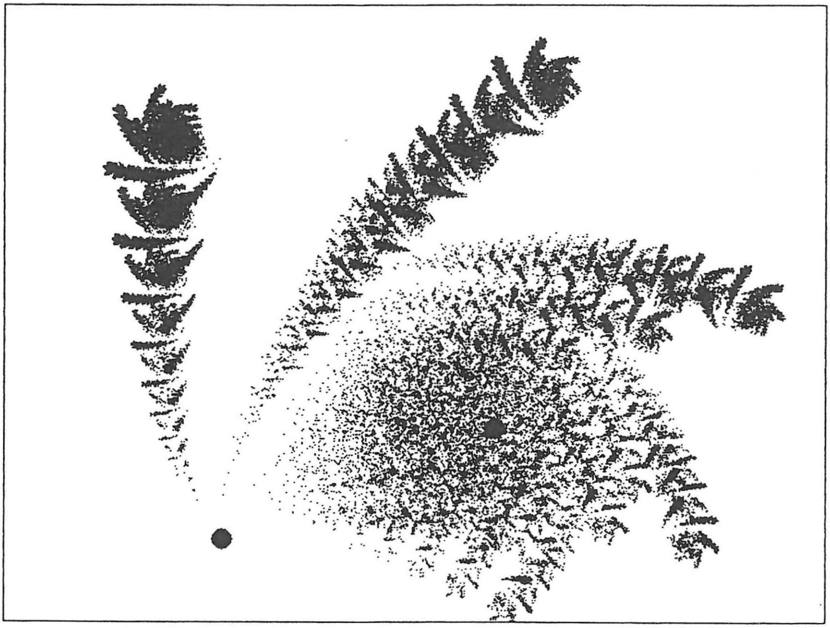

Figure 5. A synthetic fractal

general, of course, in order to be able to specify computational procedures utilizable by particular disciplines. We postpone these difficulties, however, by concentrating on the very far-reaching lambda-calculus in its form as the essential computational engine of the programming language LISP [52]. Because LISP is very common in the discipline of Artificial Intelligence (AI), we will at the same time make accessible a considerable stock of valuable tools to triadic semiotics [53].

The basic syntactic construct of LISP is the list, consisting of atoms, which are elements undivisible yet able of possessing complex features, as well of LISP-lists again which are in this case called sub-lists. Truth-values, numbers, names and certain texts can be handled as atoms. Now, by a mathematical function d, unspecified for the time being, triadic signs and sign-schemata can be mapped to LISP-expressions:

(1)	S = (M O I)
	d d d d
(1a)	ds = (dM dO dI)

Let us call the sign ds the 'cognitive descriptor' of the sign S. Thus, we are interpreting dS as a special symbolic description of S. This holds good respectively for dM, dO, dI. The descriptors dM, dO, dI can of course abbreviate complicated structures comprising atoms or lists as their sub-expressions, particularly sub-lists of the special form

(2) (LAMBDA (... x ...) (... x ...))

The descriptor-forms in (1a) are nothing else than lambda-expressions or abbreviations thereof; for an example, see the function add in (6). With the aid of the syntactic and semantic capacitiy of this formal architecture, the tool of cognitive description acquires the computational power of the Turing-machine [54]. This is necessary by all means, if signs shall become active as agents. Lambda-expressions (2) are easily comprehensible if one carefully studies the following examples of definitions of sample numerical functions which are to be recognized as syntactically different, though indeed as computationally completely equivalent:

(4)	Infix-notation:	add(x,y) = x + y
(5)	Prefix-LISP-notation:	(add x y) = (+ x y)
(6)	LAMBDA-LISP-notation:	add = (LAMBDA (x y) (+ x y))

Here, the lambda-expression (6) differs from (5) essentially by explicitly getting along the list (x y) of its variables. This feature, however, is safeguarding the machinery of substitution when the function is to be applied, as in the example

(7) (add 3 4) = ((LAMBDA (x y) (+ x y))(3 4)) = (+ 3 4) = 7

Formula (8) provides for a cognitive description dQ of square number 1 in figure 1. There, the now specified symbol d appears in a horizontal notation, elucidated by an arrow; of course, this style of notation will not change the meaning of d:

(8) (M O I) -d-> (((0 0)(1 0)(1 1)(0 1)(0 0))
universal-square "Square") = dQ

The descriptor-list dQ contains three sub-expressions: 1. A description of the sign-medium given by a list of five coordinate-pairs, in turn denoting the vertices of the closed boundary path of the square; 2. A reference 'universal square' to a geometric object; 3. The meaning-giving text "Square", providing for an interpretant. Thus, dQ specifies a complete triadic sign.

Another semiotically very meaningful possibility with cognitive description of signs is the introduction of external contexts serving as extensions of the interpretant. For instance, in the text-generating function (9) the symbol '+' denotes the operation of concatenation of texts:

(9) (LAMBDA (b) (+ c b "square"))

Now, in (9) c is a so far completely uninterpreted free variable. However, the value of c can be fixed by a 'higher located binding' of c, e.g. to the colorful symbol "red" chosen from an interpretative context:

(10) ((LAMBDA (c) ((LAMBDA (b) (+ c b "square")) "big")) "red") =
((LAMBDA (c) (+ c "big square")) "red") = "red big square"

Thus, the global contextual introduction of "red" in this example illustrates a principle of 'wide angle control' of semiotic structures.

How efficient is the LISP-apparatus of cognitive description really, when it comes to working on hard and complicated problems? We are attempting to supply a general answer which is nevertheless aiming at a 'triadic approach', providing for starting points of effective problem-solving [55]. 1. The medium-descriptor dM may give a complete declaration of the framework of the sign, including procedures for manufacture, servicing and restoration, down to the specification of raw materials; 2. The object-descriptor dO may seek for and find the wanted object itself or at least cues to it in data bases; it can even start design and constructional processes of object-creation; 3. The interpretant-descriptor dI, finally, may by examination of further data bases fix or supplement the exact meaning of the sign.

A semiotics-oriented 'characteristica universalis' of this kind will naturally call for computer power as big as possible. For instance, the interesting problem is posed, of distributing computations separately demanded for by each of the descriptors dM, dO, dI, to distinct computing modules, say CM, CO and CI, working simultaneously and supplementing each other [56].

We have realized by now that the description d is essentially a function, mapping one sign S to another sign dS. To be sure, d must fulfill a certain deep-rooted condition to be explained at once. At first, we assume that the arrow --s--> in (11), signalizing certain transitions of a sign to a next one at times, represents semiosis, stepwise producing a final sign sn = S:

```
(11)      s1  --s-->  s2  --s-->  ...  --s-->  sn  = S
          d           d                        d     d
          ds1 --ds--> ds2 --ds--> ... --ds--> dsn = dS
```

We assume, furthermore, that dS = dsn can be stepwise realized with the help of machine-based LISP-operations ds. With other words, there are phases ds1, ds2, ..., dsn, where each dsi is a computational representation of si, e.g. a datum going to memory, or a picture of si displayed on a television screen. For every i from 1 to n, si is by d mapped on dsi. Now, the following statement must hold true: Any interpretation of the complete scheme (11) along a path leading from s1 to a si, then turning down via d to the picture dsi of si, then again horizontally proceeding to dsn = dS, is independent of the special place or index i. That means that in any phase of the genesis of the signs sn and dsn we are allowed to change from the declarational viewpoint to the computational viewpoint.

The just stated property of the cognitive description d is called the 'commutativity' of (11), because the vertical places of possible change of the interpreter's viewpoint are commutable, i.e. interchangeable. In no way, namely, must any discussion of the evolutionary genesis of S depend on the phase, where we are changing from the use of signs si to their descriptive counterparts dsi.

The consequences of commutativity are very significant. They confirm the signs si as elements of a mathematical category, the signs dsi as elements of a second such category, both categories serving as tools of the science of semiotics [57]. In the mathematical theory of categories the scheme (11) is called a 'commutative diagram'. Moreover, the horizontal arrows --s--> and --ds-->, which can be but need not be mathematical functions, are called 'morphisms'. Finally, the function d is defined to be a 'functor' between both categories. Thus, any cognitive descriptor can safely be considered to be a mathematical functor [58].

4.2 How humanlike can an agent be?

In section 3.2. we characterized semiosis as a concurrent cooperating game of agents. From now on, we will assume that for every agent a cognitive description exists. Then, in order to start a semiotic game, it will in many cases be convenient to be assisted by a special agent df which we will call an organizator. More precisely: If e.g. da1, da2, ..., dak are prospective game partners, then in the very beginning these signs are in 'stasis'. Therefore, they obviously are in need of an organizator df which is able to ignite the 'big bang' of the game by use of a context like (1). In this expression the signs Mo and Oo can be useful as additional constraints or sources of data:

(1) (Mo Oo (df da1 da2 ... dak))

The organizator df must behave as neutral as possible, in order not to disturb the deterministic or aleatoric operations of the agents taken care of by him.

To be sure,. by presenting the universal mechanism (1), we are unavoidably obliged to join the discussion of the question "How humanlike can an agent be, if it has been modelled by cognitive description?" This item is the more urgent, because already in chapter 3 we have permitted man-signs to be partners of equal rights in games of any agents. Besides that, there are AI-activities aiming at the design of 'autonomous systems' which very definitely must be considered as agile and intelligent robots [59]. The relations between autonomous systems and our agents are straightforward: Either an autonomous system can be redesigned by use of advanced LISP-technology [60], or cognitive description is used as a standard specification method for remodelling the system into an agent-game.

Frequently and absolutely justifiable, the discussion will circle around the concepts of intentionality and consciousness [61]. Then, it will at least prove to be very difficult to deny agents intentionality if this quality is understood strictly as teleological behavior. This will be the case anyway, if goalseeking is considered as a problem-solving procedure. When, for instance, one gives this order to our agent A: "Try to solve problem p!", then A may figure out a strategy, may however at the same time compute a probability of success of somewhat less than 1. Accordingly, it may say: "I guess that I can solve p, so I'm going to get to work now." Thus, A is 'obviously' intending to solve p.

Somewhat easier to decide is the question whether our agent A does or does not act consciously. We assert that A does, however, only during a state where some man-sign H has adjudged this very quality of consciousness to that momentaneous state of A. The proof of this proposition must begin with the presupposition that the design of A is including the ability to communicate by common speech. Moreover, A shall posess a security system which is alerting A, when certain areas of its body are under threat or even attack. Then, it must be in the interest of fast fault-diagnosis by A itself or by any of its partners, that A is able to signalize its momentaneous state by utterances of the sort "I have a severe headache". Then, H will have no reason to doubt that A is indeed in trouble and needs help, perhaps at once. On the other hand, H must never forget that utterances by machines in general can be extremely alluring, which fact has sufficiently been proven by the famous even if simple computer program ELIZA [62].

One might be inclined to believe that the human designer H of an agent A should always be able to survey every single state of A where A might refer to sensations. This, however, need not be the case, because A could have succeeded in the course of time, just by reason of its design as well of its experience, to develop complete varieties of innovative states which might or might not ever become evident to an external observer. Here, we will adjust to the custom to call the encompassing set of the possible states of a system its phase space. Then, one can try to establish sensation-oriented linkages between human and non-human agents considered as systems. Thus we define:

(3)	Sensations:	sh1, sh2, ...
	Human states:	h1, h2, ...
	States of machines:	m1, m2, ...
	Pseudosensations:	sm1, sm2, ...

Then, it will at least in some areas of some phase spaces be possible to concretize these above-mentioned sensation-oriented linkages between H and A by a commutative diagram:

```
(4)     sh1 --sh--> sh2 --sh--> ... --sh--> shn
        B           B                       B
        h1  --h-->  h2  --h-->  ... --h-->  hn
        T           T                       T
        m1  --m-->  m2  --m-->  ... --m-->  mn
        P           P                       P
        sm1 --sm--> sm2 --sm--> ... --sm--> smn
```

Let us illustrate this rather formal-looking framework of morphisms and functors with a simple example: H as well as A shall possess highly developed capabilities to inspect their spatial environment by vision and mutual communication. At present, they are investigating a weird ruin. Just in the moment of having inspected a gloomy chamber, they together are stepping into light. A says: "well, this is rather more comfortable." Human H, experiencing the same sensation, will now try to interpret the behavior of A. H does this by using the functor B to map its own change of sensation from dark to bright, let it be symbolized by shi --sh--> shj, to a physiological state change hi --h--> hj of its body, but proceeding at once along the linkage-functor T between itself and A to the morphism mi --sm--> mj. H may do so safely, because it is very well known to it that it was the task of T to model exactly the close sensational kinship between it and A. Now, H is quite sure that a certain module of A must have registered the change from a bad to a good illumination of the environment, thus H can proceed to morphism smi --sm--> smj, interpreting this one as the cause of the utterance of A: "Well, this is rather more comfortable." Hence, H will not have the least reason to deny consistency and correctness to the communicative behavior of its partner A.

It should have become evident by now what the meaning of the gridwork (4) really is, namely a detail of a complete cognitive description of agent A. The strong conjecture that (4) can be rewritten to comply to triadic LISP-syntax, should not come as a surprise, even if the full proof is not yet brought forth. To be sure: It will need a huge amount of such details to specify a truly high-developed agent. Moreover, any detail must afterwards be implemented as a module, be it a material part of a computer or a soft computer program.

What we finally have won are the contours of cognitive semiotics. Of course, the technique of cognitive description can be applied to human beings as well. Then, the adequate answer to the question "How humanlike can an agent be?" can only read: "The likeness must be measured by the preciseness and completeness of the cognitive description." In order to satisfy Leibniz, the achievement of the necessarily high exactitude should really tax the intellectual power of man. What, however, must never be forgotten: It is only a pseudosensation a mechanical agent can have, and the same agent is showing as a conscious being only when thus experienced by man. Also, there is an important corollary to this statement: When we speak of the human brain as an object of cognitive semiotics, then consciousness can not be in the brain. Rather, it must be of quite another semiotic quality; certainly it has to be of Peirce's thirdness.

5 Cognitive esthetics

5.1 Bense's esthetificator

Let us not only return to the genex, i.e. the semiotope of all geometric esthetic signs, but let us try to find a sound transition from cognitive semiotics to cognitive esthetics. If the beholder's experience is concentrating on the perceptible sensoric structure of a visual sign, which structure is supported by the sign-medium, then the beholder is given the chance to approach the very core of esthetics, namely self-reality. Of self-reality Bense simply says: "It denotes the fact that the sign as such is denoting itself", although he adds to that passage a list of "originary esthetic structures", e.g. from painting and sculpture, which he intends likewise to be included in the realm of self-reality.

For a check of Bense's reflections on his central concepts, we choose square number 6 in figure 1 as an extremely simple example. This sign was from the beginning intended to be self-real, which fits to its being called "square, auto-referring ornate" (2.2). Hence, 'esthetic stripping' of this as well as any other sign, aiming at the divestment of all external veils and hulls of nonesthetic utility, must always result in the very sign again. However, esthetic stripping will raise an earnest problem: Is one really intending to destroy the original structure of the sign, or would one rather see it safely stored away for eventual later use? Considering, for instance, square number 8 in figure 1, we remember the object of this legible symbol being the science of geometry. Yet, with respect to this sign a situation can easily be imagined where the meaning of the exclamation mark has been totally forgotten, so that self-reality is almost automatically taking effect. Could not the recall of the former object be desirable?

Passages in Bense's opus do exist, which can be read as hinting at a concept of a more overt significance of the esthetic information [63]. Let us, therefore, introduce the concept of a 'semantic corona' of the esthetic sign [64]. The semantic corona of sign S should be understood as a particular sight of the "Interpretantenfeld" [65] of S under esthetic criteria of selecting connotational features. For a simple sign, we can attempt to perceive Leonardo da Vinci's 'Mona Lisa', as 'Just a smiling female beauty'. However, there are many intriguing connotations, of course, such as the harassing question "What or whose woman or friend where and when - and why destined for that graceful show?"

If interpreting the corona-metaphor verbally, the semantic corona will be the more prominent, the more pallid the light of self-reality is. Conversely, if self-reality is shining brightly, the corona will perhaps be diminished down to extinction. It is the beholder's opening widely to this very brightness which is to be recognized as the meaning of excercising 'esthetic intention'. However, the former dilemma is ever not yet solved, of course: Should the corona be destroyed in favour of self-reality, or should it be held memorizable?

But there is a unique categorical solution to the problem, sketched by Bense himself: One only has to define adequate functors, especially an appropriate forgetful functor [66]. Let the functor --BAe-->, which is forgetting denotation, i.e. consequentially excising every non-esthetic link from any sign, be called 'Bense's esthetificator'. Of course, Bense's esthetificator is cancelling the object and at the same time generating the self-reference of the self-real:

(1) S = (M O I) --BAe--> A = (M A I)

However, by the application of cognitive description, one can have conserved every original denoting information, given by or contained in the object-reference O of S. This is, of course, easily accomplished by simply inserting the object-descriptor dO to a sub-list, together with the original interpretant-descriptor dI, thus producing a new interpretant-descriptor (dO dI):

(2) (dM dO dI) --dBAe--> (dM SELF-REAL (dO dI)) = dA

Now, in the course of any semiotic game the special atom SELF-REAL will signalize self-reality of the sign dA it is part of. With other words, the atom SELF-REAL is saying to its computational environment: "Pay attention, I'm the very object referring to a self-real sign!" (see formula 4.2(1)):

(3) (Mo Oo (df (dM SELF-REAL (dO dI))))

Moreover, the game-process can realize that the interpretant (dO dI) of dA is a cognitive description of the semantic corona of S. Thus, the extended functor --dBAe--> is a corona-preserving esthetificator, yet not a forgetful functor anymore. Utilizing a by now well-known example (see figure 1), a special case of (2) would be

```
(4)    (to-screen(square 8) geometry
         "Symbol of the science of geometry")
          --dBAe-->
       (to-screen(p) SELF-REAL
         (GEOMETRY "Symbol of the science of geometry"))
```

Here, to-screen is a function, designed to channel information p describing square number 8 in figure 1 to a sheet of paper or a computer screen. When the square plus exclamation mark appears on the screen, the apt user will be able to read it and click it on, activating the program GEOMETRY to be explained at once. What is namely wanting now is a cognitive description of the object of the original sign which was just the science of geometry, as we remember. Thus, let us consider program GEOMETRY to constitute a powerful as well as intelligent data bank, providing nothing else than as much as possible of the present body of the science of geometry.

For a more design-oriented application of corona-preserving esthetification let us imagine a colored pattern S, considered as truly self-real, although to become semantically analyzed as well as visually presented by an organizator. After corona-preserving esthetification has been settled, any organizator supervising S will still be able to solve its operative task because it is allowed to derive from the list (dO dI), for example, the following information: S represents a certain mathematical surface which is to be viewed as a painted concrete ramp, leading from one floor to the next inside a huge hall of a certain design. Thus, the interpretant (dO dI) is in this context serving as a local data bank. Besides that, an expression of type (3) may contain free variables which must, of course, have been bound at an earlier moment of time and at a higher level of interpretation to global values of an even more remote outer context (4.1(10)). These values could, for instance, supply for data of optical illumination of the scene, part of which is the colored pattern.

From the preceding reflections we can derive a general statement concerning a substantial concept: 'Cognitive esthetics' must be considered as nothing but the cognitive semiotics of the selfdenoting self-real signs. Therefore, every sign does either already belong to the cognitve-esthetic semiotope, or can be included in it by some esthetificator. Whether or not in this context interpretative accesses to the semantic coronas of the signs are adequate and legitimate, will surely depend on the momentaneous situation of the semiotic game in progress.

5.2 Constructive-concrete art and the problem of the beholder

Geometric signs are ubiquitous in constructive and concrete art. We will not consider concrete art as separate from constructive art, but agree with the term 'constructive-concrete' [67]. Bense had aligned himself very definitely to the historical trend of the constructive-concrete, when in part I of his "Aesthetica" he cited and analyzed remarks by Max Bill [68]. According to Bense, the principle of concrete art is realized by Bill as the concretion of certain courses of abstract thinking; in this way, for instance, a red point would express its artistic reality just by its being related to the plain surface. Bense reduced this notion to semiotics by interpreting the relation of the point to the surface as an interdependence of signs. Likewise, he interpreted Kandinsky's great endeavor as work on a "sign language of pure colors and forms", even as the experiment upon an "artistic characteristica universalis" [69]. The consequences of Bense's analysis are indeed far-reaching: Any artistic production considered as a sign is an esthetic construct as well as a semiotic fact.

Returning to the ubiquity of geometry in constructive-concrete art, a serious question arises: Where will be the adequate residence of a universal geometric object O, say the square, when the beholder is concentrating on a particular materially supported piece of concrete art, where O is nevertheless 'showing'? There is the outstanding example, namely the 'Black Square' by K. Malevitch. The thought is somewhat bordering on the ridiculous, someone could take this picture for a prototype of that particular yet ideal rectangle which is distinguished by four equal sides. As a matter of fact, a sound brake, controlled by the beholder, will be preventing the automatic 'switching through' , i.e. the event of the geometric bypass (3.2), to any geometric universal.

However, when scrutinizing statements by artists and art scientists on the rôle of shape, one will very often find the concept of the geometric object in no way cautiously separated from the concept of the medium. This ambiguity is also conspicuous in the discussion of Bill's famous paper "Die mathematische Denkweise in der Kunst unserer Zeit" [70]. When Bill says that the mathematical way of thinking is not to be confused with mathematics as such, he is precisely pointing out the difference between the mathematical object on the one hand, the self-real geometric work of art on the other hand. From this, a postulate can be derived: "The presence of the mathematical object in any work of art must not be 'buoying up' but rather 'assisting' the act of beholding. When resorting to the central topic of section 5.1, one may conclude that some semiotic representation of the mathematical object should just be part of the semantic corona of the work of art in question. Let us, therefore, speak of the 'inverse bypass' when we are meaning this shifting of the center of gravity from a geometric object to its esthetic 'filial signs'.

To be sure, the phenomenon of the inverse bypass will raise further problems. This is true mainly because colors and shapes are perceived differently by different beholders. Even the following consequence cannot be simply denied: "The picture originates in the beholder" [71]. There will presumably exist consent about the equivalence of, for instance, these two statements regarding a certain painting by Kandinsky: "I'm perceiving this painting in my way" and "When I behold this painting, I'm having my own image of it". We rely on this assent, when following Mahlow's usage of the term 'mental image' [72]. Consequently, the mental image should be understood as a picture created in and by the beholder, as a true synthesis from originality, reflection, conscious processing.

Obviously, in literature on visual perception the term 'mental image' is not much in use [73]. There, "to have a mental image" will usually just mean "to perceive consciously". However, mental images must be considered to constitute substantial aspects of consiousness. Now, in section 4.2 we decided that the consciousness of man cannot be in his brain. Consequently, the question put earlier is complicating: "What then is the residence of the geometric object, as well as of the mental image?" There is only one way out of this difficulty for the advocate of cognitive esthetics: One may safely make statements of mental images if one does not keep aloof of chances of paralleling these statement by sound cognitive considerations and constructions [74].

When really dealing with mental images, we will have to reconsider the before-mentioned esthetic filial signs of geometric objects. Having adopted the concept of the mental image, we may conclude that any esthetic filial sign can be related to mental images. Of course, the filial sign can as well be a material thing as a description of that thing, e.g. a painting by Kandinsky or the sentence "My impression of that Kandinsky". Max Bill, as well, when he speaks of an 'abstract thought', namely the mental image of a red point as related to the plane, cannot be far of mental imaging [75]. He, however, is willing the thought to become concrete: It is a generative and creative process which unfolds the single mental image into a pictorial world. Obviously, this is also the meaning of Mahlow's term 'opening of the space of imagination' [76].

When again resorting to Bense, then in his information-theoretic framework a stable seat for the mental image is localizable. This place will be marked by the concept of 'esthetic communication', the scheme of which is given by an 'expedient' transmitting signs Se to a 'percipient' [77]. According to Bense, any generative process resulting in a Se must have access to a repertoire Re from which sub-signs are to be selected; likewise, any message-decoding process resulting in a sign Sp at the recipient's end must be able to select from a repertoire Rp. However, there is neither guarantee nor necessity of Re and Rp being equal. Then, the solution to the question of the semiotic positions of mental images Me and Mp, tied respectively to the signs Se and Sp, is the inclusion of adequate interpretational information referring to Me and Mp in the interpretants of both signs. In case cognitive description has been accomplished, the mental images will correspond to activated 'traces' or memory contents or picture-supporting agents of a still future design. This will certainly be the case when we are dealing with filial signs of geometric objects.

We have accepted by now that any communication in progress can be interpreted as a game of communicators or agents. Then, the communication scheme for two or more communicators can be cognitively described by our general formula

(1) (Mo Oo (df da1 da2 ... dak))

from section 4.2. In recent years, computer science has developed an approach to the organization of data exchange, called 'object oriented programming', which can with advantage be applied to our scheme (1) [78]. Because with this information-processing style, the term 'object' is fundamentally referring to bundles comprising data together with operations working on them, we will in this context rather speak of 'd-objects'. Among the procedures of operating owned by any d-object, there will almost always be messages to be passed to other d-objects. By message-passing a d-object can ask other d-objects for data or even cause them to do something. Now, to any two agents dai and daj from (1) the status of d-object can be conceded, because agents generally allow the redefinition as d-objects [79]. Therefore, message-passing between dai and daj establishes the method by which agents dai and daj communicate in their respective rôles as game partners.

5.3 Geometry from the machine

Up to now, cognitive description has been presented as a tool for the depiction of the world of the genex and the games played therein. One must, however, be aware of the fact that a formidable and still swelling stream of geometry-related entities has its origin in the market of communication technology and the worldwide machinery of the data services. This stream is transporting essentially two kinds of things and signs: 1. Technical equipment for the daily use in research, industry, administration, defence, household; 2. Signs migrating in the medium, i.e. the compound of television and the other public data channels.

Entities of the kind just mentioned originate from a process which through its acronyme 'CAD' i.e. 'Computer Aided Design', has become the outstanding paradigm of technical creation [80]. Part of the self-appreciation of CAD is the motivation to optimize the esthetic as well as the functional modelling of its products. This principle is furthering the unending expedition of geometric-esthetic signs distinguished either by the fascinating forms of mechanic and electronic gadgetry, or by the elegant shapes of its casings. CAD is heavily assisted by the rather young science of 'computational geometry' [81].

When we inspect the worldwide electronic medium, two important expedients of signs make themselves prominent: Advertising and show-business. Here, the beholder will in future have difficulties, should he try to differentiate between signs originating from human work or else having been designed and put out completely by machine [82]. 'Computer graphics' is the huge scientific [83] and technological as well as esthetic [84] domain promoting this process by providing for tools of growing power.

CAD and computer graphics have evolved their own computational traditions and languages besides the LISP-oriented symbol-processing culture of AI, although recently confluence is identifiable. However, AI and LISP can always be resorted to as a universal method of assessing as well as re-thinking and possibly augmenting the methodologies of other fields of development. Moreover, one cannot say whether very advanced LISP-computers will not possibly be the universal calculating machines of the future. What is of even greater importance: Cognitive description in the sense of a 'characteristica universalis' may be the only key to a fundamental comprehension of the medium-supported dynamic agent-game we are all partners of. To be sure, such in-depth understanding will be much in need of help by still other sciences. There, a new science is having its début these days which could prove very useful: 'Dynamics of complex adaptive systems'. Key notions of this science are 'chaos', 'self-organization', 'adaptation' [85].

5.4 Virtual reality: Geometrized worlds

There is an enormously wide scale of luxury or otherwise scarcity of the common spatial visual ambiente man is accustomed to. In any case, however, there will be geometric redundancy around the beholder, showing on or in the depth of solid surfaces of material bodies. Quite another perceptive context, however, is generated by looking into a stereoscope. What one can see there, might as a matter of fact have been created by computer. If, now, the stereoscope is of such a perfection that the observed pictures come not only alive as in a movie, but are changing in a completely normal way when one turns the momentarily used stereoscope around like a common binocular, then one can rightfully speak of beholding a 'synthetic world'. Computer scientists who have developed such a contrivance prefer the terms 'virtual reality' or 'virtual world' [86].

Virtual reality is generated as a multimedia stream of information, originating in a powerful computer. The mentioned stereoscope, now generally called a 'head mounted display', does not only comprise tiny picture screens, one for each eye, but is augmented by binaural earphones. Even more important: The haptic sense is included in virtual reality by 'data gloves' and even 'data suits' which not only feed complex information on the wearer's movement to the computer, but channel information on the 'solid' structure of its virtual environment back to it [87].

If, some day, computers powerful enough for virtual-reality-technology will become as inexpensive as the desktop computers are today, then an innovative structuring of social life should be attainable because the real world and its virtual counter-ambiente can in fact be superimposed and even blended [88]. Then, the beholder can communicate with 'virtual partners', including itself but 'virtualized', as well as with a real human partner really beside it in 'olfactory reach'.

Virtual realities must be considered as recently immigrated though prominent citizens of the domain of the genex because anything one can perceive in a virtual ambiente is a sign, generated according to form-bound laws of redundancy. One can even think of the future profession of a 'virtual-world-designer [89].

5.5 Pygmalion and the question of survival

Why does the creation of a perfect robot, of an ideal counterpart of man, define a goal which appears seductive and at the same time absolutely foolish? Does such a wish not range with multiplication of the lifespan of the human individual, or the arriving at an alien civilization in outer space? Or do we meet here the hard core of an essential question? In defense of the dream, one should mention that doing work on borderline problems of this sort could serve one common goal: To learn something important about the nature of man [90].

As a matter of fact, there has never been any stop of the effort of creating artificial man. However, engineers and artists are approaching the goal on different paths, even if these are parallel at times. In section 4.2 we already discussed the question how humanlike an agent can be. There, the essence of our answer was: "To an amazing extent." We also mentioned 'autonomous systems' which are the object of research and design done with endurance. In art it is the high geometry of the human body which has always stimulated phantasy and creative power: Everyone knows the myth of Pygmalion, sculpturing a lovely girl to become alive by the judgment of the goddess; but there are qualified artists like Albrecht Dürer and Oskar Schlemmer to carry on the tradition.

Thus, artificial man must be considered as posing the 'horizon-problem' of cognitive esthetics. Its creation in a hi-tech-laboratory would afford the cooperation of specialists of very many fields. And knowledge, experience, ability of those artisans and craftsmen would have to be focussed into one single goal: To get this manlike artwork moving, going, speaking, reasoning, helping, serving [91]. The scenario depicted is anyway sharply contrasting to the too often jungle-like teeming of erratic events, displayed by the genex today. Paradoxically, artificial man, deliciously as well as athletically moulded perhaps [92], would, on the other hand, as a rare piece of self-reality doubtlessly comply with Bense's saying: "In principle, the esthetic information, hence the work of art realized, is unimaginable prior to its factual production. Moreover, its reality is of the utmost fragility, brittleness. The potential chances of its becoming suspended, destroyed, are enormously large" [93].

Remarks

1 Peirce Coll. pap., 8.327. Peirce 1965.
2 Bense 1992, p.71.
3 Bense 1967, p. 16.
4 McCarthy.
5 Peirce Coll. pap., 8.333f.

6 Bense 1992, p. 75.
7 Bense 1965, p. 321. Bense and Walther 1973, p. 45.
8 The exposition to follow owes very much to E. Walther's brilliantly written introduction to Bense's semiotic and esthetic opus: "Max Benses Wendung zur Semiotik" (Walther 1991).
9 Bense 1939. Bense 1946 (1949).
10 In this paper translations of statements by Elisbath Walther, Max Bense and Dietrich Mahlow into the English language are by Nees.
11 Bense 1965.
12 ibid., p.320.
13 ibid., p. 333f. Bense and Nees 1965. Bense 1969, p. 62.
14 Bense 1969, p. 56. Gunzenhäuser 1962.
15 Bense 1965, p. 282; 1969, p. 56.
16 Bense 1965, p330.
17 Bense 1967; 1971; 1975; 1976; 1979; 1981; 1983; 1986.
18 Bense 1992, p. 75.
19 ibid., p. 75. Bense describes self-reality as follows: "It designates the fact that the sign as such denotes itself. Furthermore, I have demonstrated that this is not only valid for the sign as such, but for the number as such and for the works of art as such, or else for original esthetic creations (painting, sculpture, poetry, prose, musical compositions or stage plays etc.)."
20 Lorenzen, p. 164.
21 Hargittai.
22 Minsky. From this book of the AI-scholar we borrow the term 'agent'. According to Minsky the human mind is a society of cooperating agents. For artificial brains cf. Ashby; Drescher; Kent; Minsky.
23 Peirce Coll. pap., 6.302.
24 Peirce Coll. pap., 5.283: "When we think, then we ourselves, as we are at that moment, appear as a sign." Ibid. 5.313: "Upon our principle, therefore, that the absolutely incognizable does not exist, so that the phenomenal manifestation of a substance is the substance, we must conclude that the mind is a sign developing according to the laws of inference", and "But this consciousness, being a mere sensation, is only a part of the material quality of the man-sign."
25 Bense 1965, p. 242f. Cf. Walther 1974, p. 123; according to this text, semiosis must be understood to be "a process which is related to signs, plays on signs, or is carried on by signs". Here "to play on signs" is our translation of the author's phrase "sich an Zeichen abspielen".
26 Cf. Bense 1983, p. 81f. Cf. Leroi-Gourhan; Zur Lippe; Eibl-Eibesfeldt und Sütterlin; Eibl-Eibesfeldt.
27 Krohn und Küppers. This is a reader dedicated to the concept of self-organization. Peirce designed a cosmogonical philosophy which clearly shows

self-organizational features. Coll. pap., 6.33: "It would suppose that in the beginning - infinitely remote - there was a chaos of unpersonalized feeling, which being without connection or regularity would properly be without existence. This feeling, sporting here and there in pure arbitrariness, would have started the germ of a generalizing tendency." "Generalizing tendency", however, provides exactly for the base on which visual structures can organize themselves.

28 Bense ascribes self-organization directly to semiosis. Bense 1986, p. 124: "But now it is easily to be seen that for the introduction of a sign (Z) or a triadic sign-relation (ZR^3) in the sense of a sign class (ZKL^3), a preceding sign or a sign relation (ZR^3) or otherwise a complete sequence of signs is necessary, just as corresponding to an introduced sign-relation a complete sequence of such relations can follow. Of course, here lies one of the origins of a certain 'self-organization' of the sign, which can be receding as well as proceeding."

29 An der Heiden. Cf. Haken and Karlqvist and Svedin.

30 Dingler, Lorenzen.

31 Platon: Menon 82b-85e. Becker, p. 109f; The figure used by us corresponds to one contained in that book.

32 Platon: Politeia 509c.

33 Ibid., 509d.

34 The concept of symmetry discussed is closely tied up to F. Kleins 'Erlanger Programm' in mathematical group theory; Becker, p. 197f. Connections to esthetic harmony and proportion: Speiser; Weyl; Bense 1939; Bense 1949. Actual discussion: Franke 1995; Hargittai; Gerstner; Hardison; Hofstadter 1979; Hofstadter 1985; Leyton; Mazzola; Nees 1990; Nees 1990b; Nees 1995;Schroeder; Stewart and Golubitsky; Weinberg.

35 Hilbert.

36 For literature on modern geometric theory, see bibliographical data in: Goldblatt; Haken; Lu; Mac Lane.

37 'Mind and Brain'; special-issue 'Mind and Brain' of Scientific American 1992.

38 Bense 1979, p. 35.

39 Bense 1965, p. 333.

40 Bense 1965, p. 276.

41 Bense 1981, p. 28: "Semiotische Morphogenese", where Bense is refers to R. Thom (Thom; Lu). Cf. Bense 1979a, p. 36f. For plant-like growth see Kaandorp; Prusinkiewicz and Lindenmayer; Nees 1995.

42 Mandelbrot. Barnsley. Briggs and Peat. Nees 1988; 1992; 1992a; 1993; 1994a; 1995. Peitgen and Richter. Peitgen and Saupe. Pickover. Schroeder.

43 'Chaos order' (Chaosordnung) has apparently become a comprehensive term for different modes of chaos. Dietrich Mahlow used the diction 'ChaosOrdnung' as the title of his film dealing with presence and meaning of the chaotic and aleatoric as connected with creativity in modern art (1992). Cf. Mahlow 1992. We are generally using 'chaos' as the singular form of 'chaos order' which

coining seems to be appropriate considering the venerableness of the cosmologic concept of chaos. In a time prior to the era of mathematical and algorithmic chaos theory, Bense pointed to the importance of P. Mongré's "Das Chaos in kosmischer Auslese" (Hausdorff; Bense 1969, p. 138f, also in connection with the 'anthropic principle'; cf. Franke 1995). Bense also discussed a pre-fractal concept of chaos as a principle below structure and gestalt (Bense 1969, p. 35, p. 53), which can be understood to constitute the bottom level of all chaos orders (ibid., p. 32): "What is completely undetermined, as a 'chaos' perhaps, cannot ever be identified and fixed, at first it must, so to speak, become transformed into an even if weakly determined state, in order to become identifiable."

44 Briggs and Peat. Peitgen and Richter. Peitgen and Saupe. Nees 1995.

45 Mandelbrot, p. 208. Mandelbrot uses 'chaotic evolution' in this context.

46 Barnsley. He coined 'chaos game', what can be understood as synonymous to Mandelbrot's 'chaos evolution'. Cf. Nees 1992; 1995.

47 What one sees is a presentation of points belonging to the fractal: Every such point, considered as a complex number z, is checked for its 'phlegma' (cf. Nees 1995), if again and again transformed by the rule'next z becomes z*z-c', where c is a constant. The points of low phlegma can be observed as fleeing their starting points very fastly. Only points of highest phlegma were made visible in black color. Thus, the fractal diagram generated is demonstrating the principle 'maximal phlegma'. Analytic fractals of this kind are generally called 'Julia-sets' (Peitgen and Richter; Peitgen and Saupe; Pickover).

48 The synthetic fractal visualizes the trace of a single migrating point led across the picture plane like a ball of the game 'Boccia', where in this case exactly three players are involved; cf. Nees 1992b.

49 For the concept of cognitive thinking see the readers: Kognitionswissenschaft (Münch); Kognition und Gesellschaft (Schmidt); Emergenz: Die Entstehung von Ordnung, Organisation und Bedeutung (Krohn und Küppers). 'Cognitive' is prefixed attribute to the name of several sciences; for lingusitics cf. Winograd 1983, p. 20, who mentions the 'generative' and the 'computational' paradigm, then proceeds: "From a greater distance they may be seen as two variants of a single 'cognitive' paradigm, ...", and: "The proper domain of study is the structure of the knowledge possessed by an individual who uses a language", and: "This knowledge can be understood as formal rules concerning structures of symbols."; cf. Winograd und Flores. For psychology: Hayes, p. 1: "Cognitive psychology is a modern approach to the study of the processes by which people come to understand the world, such processes as memory, learning, comprehending language, problem solving, and creativity. Cognitive science has been influenced by developments in linguistics, computer science, and, of course, by earlier work in philosophy and psychology." Cf. the early book Betz: Psychophysiologie der kognitiven Prozesse.

50 Becker, p. 355.

51 Münch, p. 10.

52 LISP: McCarthy; Allen; Stoyan und Görz; Abelson, Sussman and Sussman. For the lambda-calculus: Curry; Stenlund.

53 Banerji. Elithorn and Banerji. Charniak and McDermott. Nees 1985; Nees 1985a.Winston.

54 Davis, p. 10.

55 Winston. Schank. Charniak and McDermott. For language processing: Gazdar and Mellish; Görz; Handke.

56 Görz, p. 13. For the hardware problem: Hillis.

57 For category theory: Goldblatt; McLane; Arbib andf Manes; Goldblatt; Pareigis. Computation of categories: Rydeheard and Burstall. In music: Mazzola.

58 Bense introduced the theory of categories and functors into triadic semiotics and referred to it in many of his papers. In Bense 1969, p. 122 he interprets Peirce's semiotic derivation of metaphors as a concatenation of morphisms. Bense 1979, p. 49; there the relation between a sequence of photographs and a corresponding sequence of paintings is interpreted as a functor. For the application of category theory to the foundations of semiotics, especially to semiotic graphs: Bense 1981, p. 139; Bense 1983, p. 66. There is a poem by Bense (Bense 1988, p. 12), the title of which refers to the concept of the forgetful functor (cf. Bense 1986): "Hadamards 'Vergiß-Funktoren'". There exist also papers on category theory in semiotics by his former students, cf. Leopold.

59 Chapman. Drescher. Maes. Wallich.

60 Hillis.

61 Münch. Schmidt. Krohn und Küppers.

62 Weizenbaum.

63 Bense 1969, p. 32: "The dialectic between physical and esthetic states is becoming visible in only very few artistic objects, because the <semantic> state of conventional determination compensates. But it is a trait of modern art production to suppress mediation and to fixate the esthetic factor as a thwart to the physical aspect. I draw attention to Kandinsky's first subjectless picture or to the results of recent electronic-stochastic techniques of composition."

64 Bense 1979b. This paper presents the perhaps most striking introduction to Bense's ingenious technique of esthetic speculation. There our 'semantic corona' is factually Bense's "creative interpretant" drawing information from the "open, surrealistic or even labyrinthic context" (loc. cit. p39).

65 Bense und Walther 1973, p. 45.

66 Bense 1986, p. 102: "In the selectively representing actions of the creatively realizing consciousness, those triple-connections are stepping forth as easily as they are receding. Obviously, the 'forgetful functor' to which we will revert and of which much use is made since the French mathematician's Henri Poincaré and Jaques Hadamard's 'The Psychology of Invention in the Mathematical Field', is also extensible beyond the mathematical domains."

67 Rave and Witt, p. 5. Cf. Rickey.
68 Bense 1965, p. 61, 62. Bill 1947. Cf. the reproduction in Frei, p. 265.
69 Bense 1965, p. 62. Kandinsky 1925; 1926.
70 Bill 1949. Cf. Bense 1979c. Nees 1991, p. 20, p. 30. Mahlow 1969.
71 Mahlow 1989, p. 51: "The perception of the beholder has been neglected. Yet he is as important as the artist and his work ..."
72 D. Mahlow has repeatedly and in a canonical way summarized procedures concerning realization, analysis and cultivation of the mental image. Mahlow 1969, p. 4: "Finding of unexpected solutions, penetrating complex relationships, taking interest in evolutions from simple origins, becoming aware of optical reactions, checking visual fundaments." Mahlow 1989, p. 51: "Some neurobiology: the movement - eye - brain - language, concept, or: eye - brain - and back to eye again - to the hand: with those who cause something."
73 Cf. the discussion of introspectionism Hayes, p. 19. Cf. the positive discussion of the 'Vorstellungsbild' Kosslyn and Pomerantz. Cf. Holenstein.
74 Mahlow 1989, p.51.
75 Bill 1947.
76 Mahlow 1989, p. 51. "Is then constructive art not more than a picture of language, of reflection? Can it open and extend the space of imagination?, explore new possibilities?"
77 Bense 1960, p. 27f. Bense 1969, p. 19f.
78 Stoyan und Görz. Collins and McMillan.
79 Hewitt. Abelson, Sussman and Sussman. Magnenat-Thalman and Thalman. Reiser und Wirth.
80 Encarnacao and Schlechtendahl. Ten Hagen and Veerkamp. Medland. Nees 1980.
81 Earnshaw.
82 Nees 1990a.
83 Abelson and diSessa. Encarnacao und Strasser. Foley and Van Dam. Giloi. Gnatz. Newman and Sproull.
84 Franke. McCorduck. Pickover. Steller. Whitney. Nees 1969; 1990; 1995. Zemanek. Zuse; for an early paper "Über den Einsatz von programmgesteuerten Rechenmaschinen auf dem Gebiet der Graphik und des Kunstgewerbes" by Zuse cf. Bense 1990.
85 Waldrop. Cf. Ruthen, p. 112: "An adaptive system, then, consists of many agents, each interacting with others that have the same or different strategies."
86 In Germany, Darmstadt is known as a surpassing center for the development of virtual reality; see Encarnacao 1991.
87 Foley.
88 Conn. Franke 1987. Nees 1990a; 1993; 1993a; 1994; 1994b; 1995.
89 Franke 1987a. Nees 1982, where H. W. Franke's eminent fictional work on virtual realities is appreciated. Cf. Lem. Nees 1994b.
90 The brillant and stimulating speculations on the robot by the late A. A. Asimov

are mentioned here with gratitude. Cf. Berry, Nees 1994.
91 Nees 1993; 1994.
92 Stapledon.
93 Bense 1960, p. 22.

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Internationale Zeitschrift für
Semiotik und Ästhetik
20. Jahrgang, Heft 1/2, 1995

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